Worksheet 11: Point estimation

Example 0.93. Suppose the weights of the 12 eggs in a selected carton are

$$x_1 = 63.3,\ x_2 = 63.4,\ x_3 = 64.0,\ x_4 = 63.0,\ x_5 = 70.4,\ x_6 = 65.7,$$
$$x_7 = 63.7,\ x_8 = 65.8,\ x_9 = 67.5,\ x_{10} = 66.4,\ x_{11} = 66.8,\ x_{12} = 66.0$$

Obviously, one can use the sample mean $\bar{x} = 65.5$ g as a single guess of the population mean μ.

- We say that $\bar{x} = 65.5$ g is a **point estimate** of μ.
- However, point estimates will likely vary from sample to sample.
- In order to study such randomness, we need to consider a random sample $X_1, \dots, X_{12}$ from the population and examine the associated statistic:

  $$\bar{X} = \frac{1}{12} \sum_{i=1}^{12} X_i.$$

  The statistic $\bar{X}$ is called a **point estimator** of μ.
- Note that point estimator is a random variable (statistic) while point estimate is a specific number (obtained through a realization of the sampling process).

Example 0.94. Suppose we draw a random sample $X_1, \dots, X_n$ from the uniform distribution $\mathrm{Unif}(0, b)$. Then the sample maximum

$$\max_{1 \le i \le n} X_i$$

can be used as a point estimator for b. Any other statistic may be used to estimate b?

Example 0.95. In the brown egg example, a point estimate of σ^2 based on S^2 is $s^2 = 4.72$. In contrast, $s'^2 = 4.32$.

Example 0.96. For a random sample of size n from the $\mathrm{Unif}(0, b)$ distribution, it can be shown that the sample maximum is a biased estimator of b:

$$\mathrm{E}(X_{\max}) = \frac{n}{n+1} b.$$

This implies that $\frac{n+1}{n} X_{\max}$ is an unbiased estimator of b (Recall that $2\bar{X}$ is another unbiased estimator of b).