

Worksheet 2: Counting

Example 0.15. A subway restaurant provides 5 kinds of bread, 4 kinds of cheese, 4 kinds of meats, and 6 kinds of sauces. In how many ways can you order a sandwich?

Example 0.16. How many different CA driver license numbers are there (1 capital letter followed by 7 digits)? How many driver license numbers have all repeated digit? All distinct digits?

Example 0.17. How many 3-digit numbers are divisible by 5?

Example 0.18. List all permutations of size $r = 3$ chosen from the set $S = \{a, b, c, d\}$. How many are there? What if $r = 4$?

Example 0.19. In how many different ways can 5 people be arranged in a row? Along a circle?

Example 0.20. Suppose you have 10 textbooks, in which 5 are about math, 3 about computer science and 2 about English. In how many different ways can you arrange them in a line to put on your bookshelf? What if you want to have the books of the same subject all together?

Example 0.21 (Birthday problem). Assume that people's birthdays are equally likely to occur among the 365 days of the year and ignore leap years. Find the probability p that no two people in a class of 35 have a common birthday (i.e., all have different birthdays). (Answer: .1856.)

Example 0.22. List all combinations of size 3 chosen from the set $S = \{a, b, c, d\}$.

Example 0.23. Consider the problem of choosing 4 members from a group of 10 to work on a special project.

- (a) Suppose two people A and B really like each other, so that they must be simultaneously chosen or skipped. How many distinct four-person teams can be chosen?
- (b) Suppose two people A and B really hate each other, so that they cannot be both selected for the project. How many distinct four-person teams can be chosen?

Example 0.24. An urn has 5 red balls and 7 blue balls. Suppose you randomly select 5 balls from the urn. What is the probability that your hand has exactly 3 red balls?

A ordinary deck of 52 cards is divided into 4 suits (heart, diamond, spade and club) and 13 ranks (2, 3, ..., 10, J, Q, K, A)

Example 0.25. Suppose you randomly draw 5 cards from a deck of 52. What is the probability that you have a

- (a) four of a kind (4 cards of the same rank, and one side card)
- (b) flush (5 cards of the same suit)