Worksheet 4: Random variables + distributions

Example 0.39. What are the outcomes of each experiment?

- Flip a coin once;
- Flip a coin 5 times;
- Toss two dice;
- Select four numbers from 1:20, without replacement;
- Toss a coin repeatedly until a head first appears.

Example 0.40. Find the range of the following random variables.

- Flip a coin once; $\longrightarrow X = 1$ (H), 0 (T)
- Flip a coin 5 times; $\longrightarrow X =$ #heads
- Toss two dice; $\longrightarrow X =$ sum, $Y =$ absolute value of difference
- Select four numbers from 1:20 at random, without replacement; $\longrightarrow X =$ maximum of the 4 numbers
- Toss a coin repeatedly until a head first appears. $\longrightarrow X =$ total #trials needed, $Y =$ #tails before the first head

Example 0.41. Determine the following events:

- Flip a fair coin once; define $X = 1$ (H), 0 (T). $X^{-1}(1) =$
- Toss two fair dice; define $X =$ sum. $X^{-1}(7) =$
- Select four numbers from 1:20 at random, without replacement; define X = maximum of the 4 numbers. $X^{-1}(3) =$, $X^{-1}(5)$
- Toss a coin repeatedly until a head first appears; define $X =$ total #trials needed. $X^{-1}(3) =$

Example 0.42. Find the following probabilities:

- Flip a fair coin once; define $X = 1$ (H), 0 (T). $P(X = 1)=$
- Toss two fair dice; define $X =$ sum. $P(X = 7)=$
- Select four numbers from 1:20 at random, without replacement; define X = maximum of the 4 numbers. $P(X = 3) =$, $P(X = 5) =$
- Toss a fair coin repeatedly until a head first appears; define $X =$ total #trials needed. $P(X = 3)=$

Example 0.43. Find the following probabilities:

- Toss two fair dice; define $X =$ sum. $P(X \leq 3)=$, $P(X \geq 10) =$
- Select four numbers from 1:20 at random, without replacement; define X = maximum of the 4 numbers. $P(X \leq 5) =$
- Toss a fair coin repeatedly until a head first appears; define $X =$ total #trials needed. $P(X \leq 3)=$

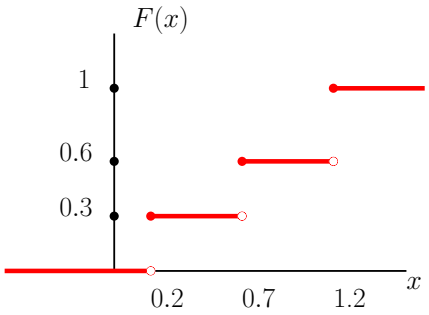
Find the pmf of X in each question below and display it in both ways.

Example 0.44 (Roll a fair die once)**.** Let X be the number obtained.

Example 0.45 (Roll two fair dice)**.** Let X be the sum of the two numbers obtained.

Example 0.46. Find the cdf of X in the top example (roll a fair die once).

Example 0.47. Find the pmf corresponding to the cdf given below.



Example 0.48. Let X be a random variable whose cdf is shown above, find

- $P(X < 0.2) =$ $, P(X \leq 0.2) =$ $, P(X > 0.2) =$ $, P(X \geq 0.2) =$
- $P(X \leq 1) =$ $, P(X < 1) =$
- $P(0.2 < X \leq 1.2) =$