

Worksheet 9: Joint distributions

Example 0.77 (Toss two fair dice). Let X denote their sum and Y the absolute value of their difference. The joint and marginal pmfs of X, Y are as follows:

$y \backslash x$	2	3	4	5	6	7	8	9	10	11	12	$f_Y(y)$
0	$\frac{1}{36}$		$\frac{1}{36}$		$\frac{1}{36}$		$\frac{1}{36}$		$\frac{1}{36}$		$\frac{1}{36}$	$\frac{6}{36}$
1		$\frac{2}{36}$		$\frac{2}{36}$		$\frac{2}{36}$		$\frac{2}{36}$		$\frac{2}{36}$		$\frac{10}{36}$
2			$\frac{2}{36}$		$\frac{2}{36}$		$\frac{2}{36}$		$\frac{2}{36}$			$\frac{8}{36}$
3				$\frac{2}{36}$		$\frac{2}{36}$		$\frac{2}{36}$				$\frac{6}{36}$
4					$\frac{2}{36}$		$\frac{2}{36}$					$\frac{4}{36}$
5						$\frac{2}{36}$						$\frac{2}{36}$
$f_X(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	

Example 0.78. For the X, Y defined above, find the following probabilities:

- $P(X \leq 4, Y \leq 2) =$
- $P(X \leq 5) =$
- $P(X \geq 11, Y \leq 2) =$
- $P(Y \leq 1) =$

Example 0.79 (Toss 2 fair dice). Suppose we are told that the sum is $X = 6$. What is the (conditional) distribution of Y (absolute value of the difference)?

Example 0.80 (Toss 2 fair dice). Find the following conditional distributions:

- Y given $X = 4$
- X given $Y = 3$
- X given $Y = 0$

Example 0.81 (Toss two fair dice). Determine if X (sum) and Y (absolute difference) are independent.

Table 2: Conditional pmfs of Y given $X = x$

$y \backslash x$	2	3	4	5	6	7	8	9	10	11	12
0	1		$\frac{1}{3}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{3}$		1
1		1		$\frac{1}{2}$		$\frac{1}{3}$		$\frac{1}{2}$		1	
2			$\frac{2}{3}$		$\frac{2}{5}$		$\frac{2}{5}$		$\frac{2}{3}$		
3				$\frac{1}{2}$		$\frac{1}{3}$		$\frac{1}{2}$			
4					$\frac{2}{5}$		$\frac{2}{5}$				
5						$\frac{1}{3}$					

Table 3: Conditional pmfs of X given $Y = y$

$y \backslash x$	2	3	4	5	6	7	8	9	10	11	12
0	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$
1		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$	
2			$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		
3				$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$			
4					$\frac{1}{2}$		$\frac{1}{2}$				
5						1					

Example 0.82. Are two random variables X, Y with the following joint pmf independent?

$y \backslash x$	0	1	2
-1	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$
1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

Example 0.83. For the random variables X, Y defined in the last example, find

- $E(X + Y) =$
- $E(XY) =$
- $E(|X - Y|) =$