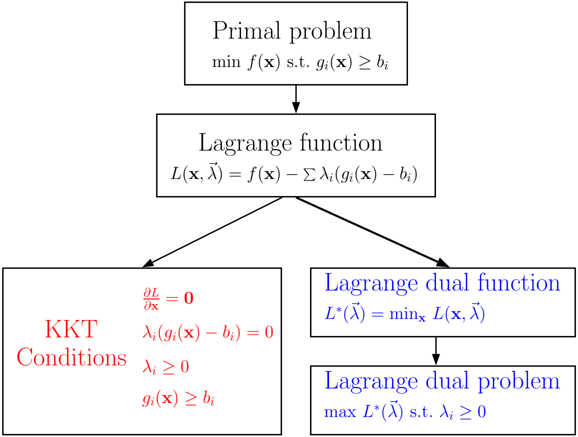
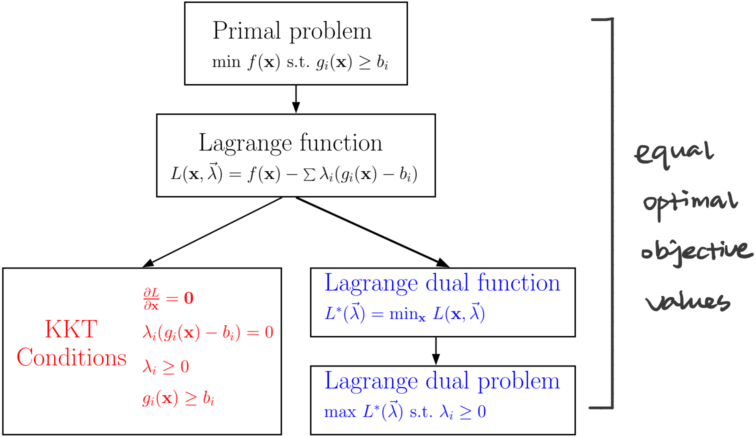
The Lagrange dual problem



The Lagrange dual problem

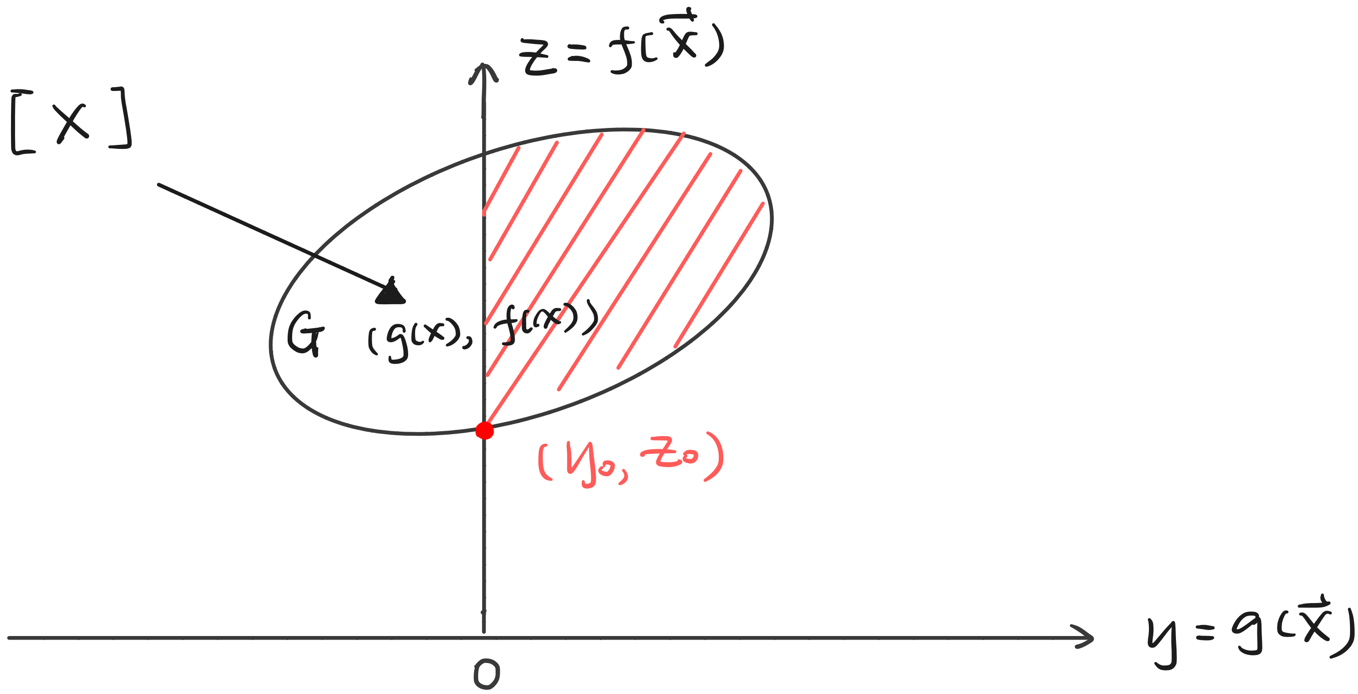


Primal problem

$\min f(\mathbf{x})$ s.t. $g_i(\mathbf{x}) \geq b_i$

Consider the following primary problem P:

minimize $f(x)$ subject to $g(x) \geq 0$



$G = \{(y, z) : y = g(x),\ z = f(x) \text{ for some } \vec{x} \in X\}$

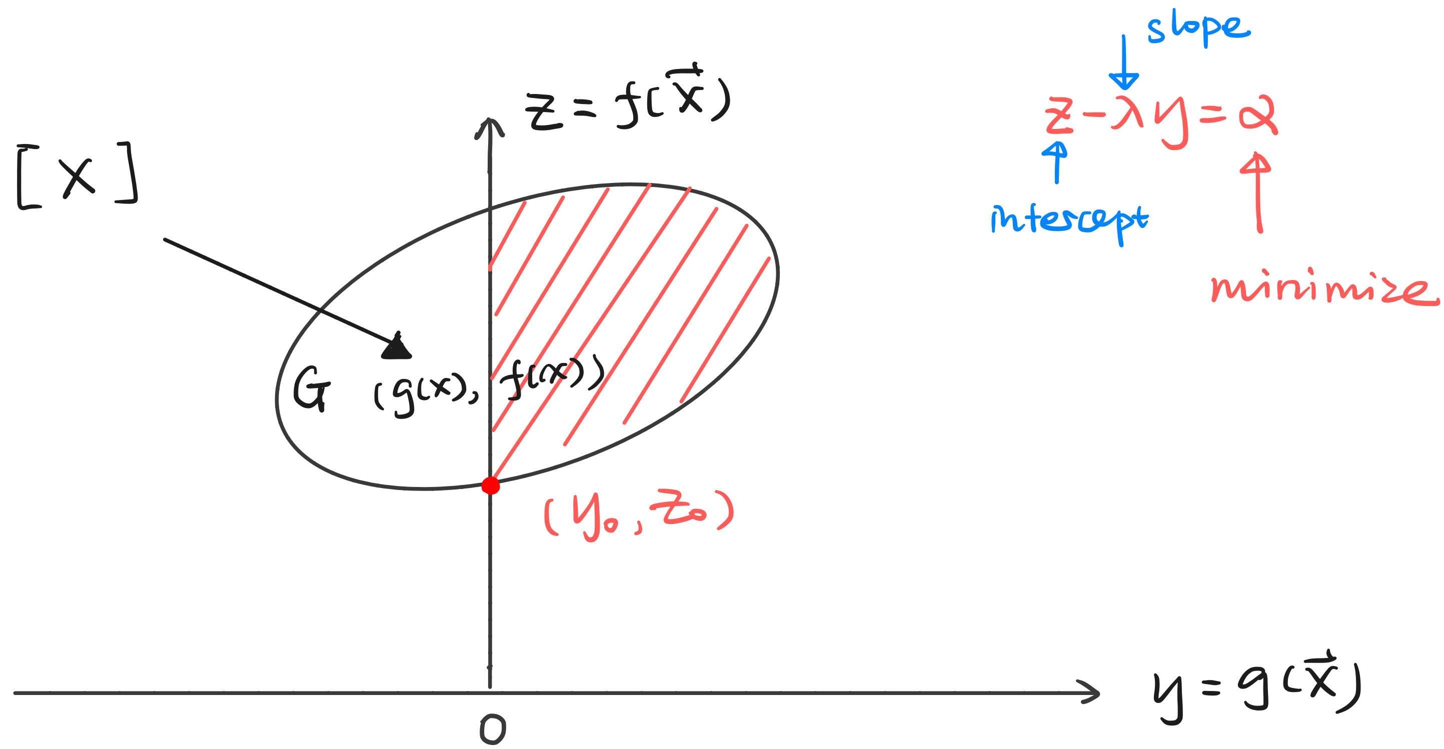
Lagrange dual function
$L^*(\vec{\lambda}) = \min_{\mathbf{x}} L(\mathbf{x}, \vec{\lambda})$

$\because z = f(\vec{x})$
$y = g(\vec{x})$

minimize $L = f(x) - \lambda g(x)$ subject to $\lambda \geq 0$

→ minimize $f(x) - \lambda g(x)$

→ minimize $z - \lambda y$ over points (y, z) in G



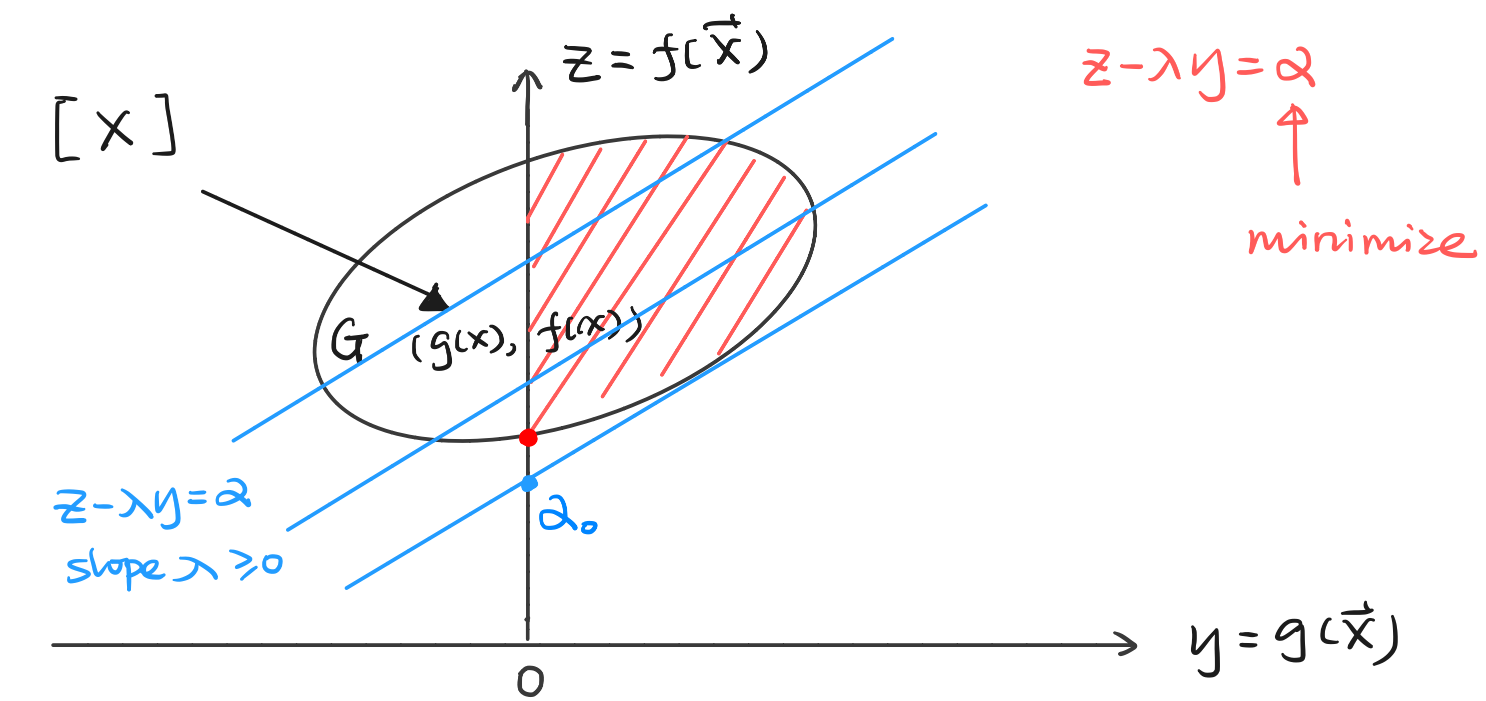
$\boxed{\begin{array}{l}\text{Lagrange dual function} \\ L^*(\vec{\lambda}) = \min_{\mathbf{x}} L(\mathbf{x}, \vec{\lambda})\end{array}}$

$\boxed{\begin{array}{l}\because\ z = f(\vec{x}) \\ \quad\ y = g(\vec{x})\end{array}}$

minimize $L = f(x) - \lambda g(x)$ subject to $\lambda \geq 0$

$\rightarrow$ minimize $f(x) - \lambda g(x)$

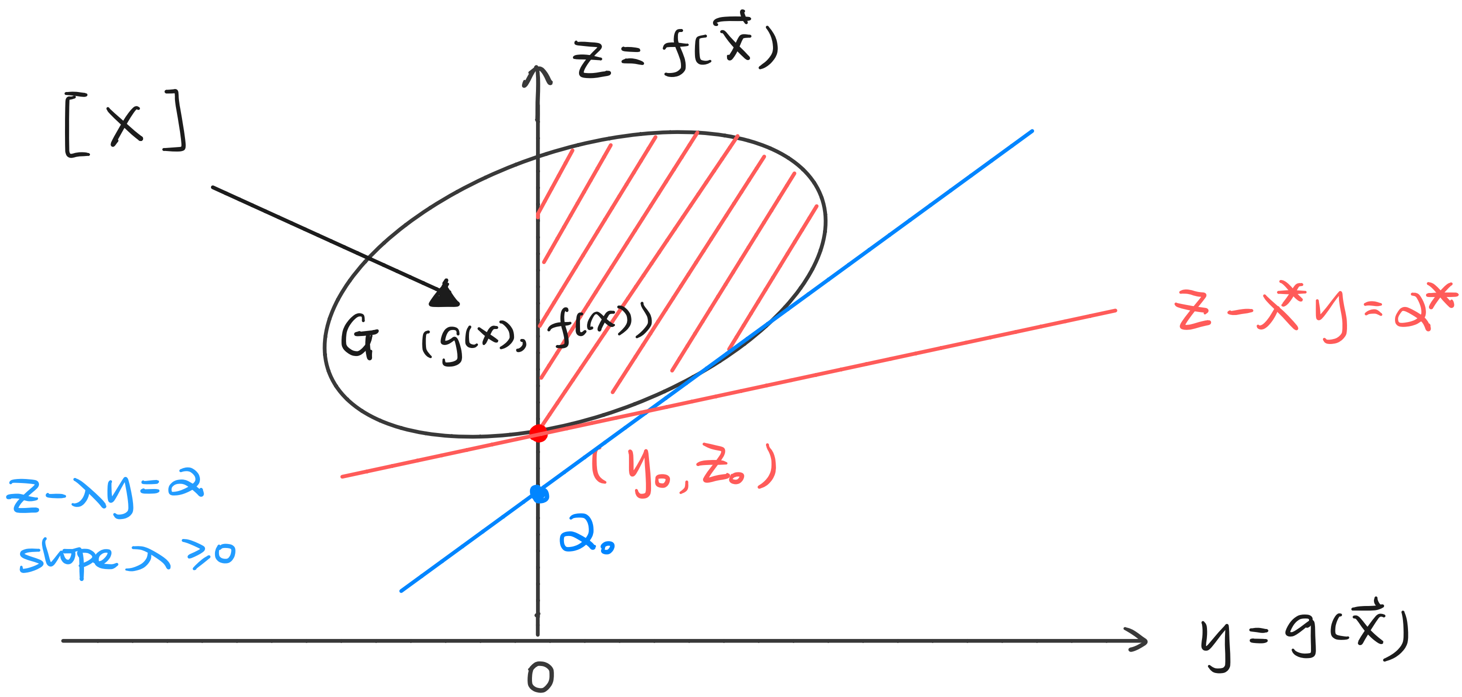
$\rightarrow$ minimize $z - \lambda y$ over points (y, z) in G



- move $z - \lambda y = \alpha$ paralell to itself as far down as possible, while it remains in contact with G
- α_0 is the minimum value of L corresponding to the given $\lambda \geq 0$

$$\boxed{\begin{array}{l}\text{Lagrange dual problem} \\ \max L^*(\vec{\lambda}) \text{ s.t. } \lambda_i \geq 0\end{array}}$$

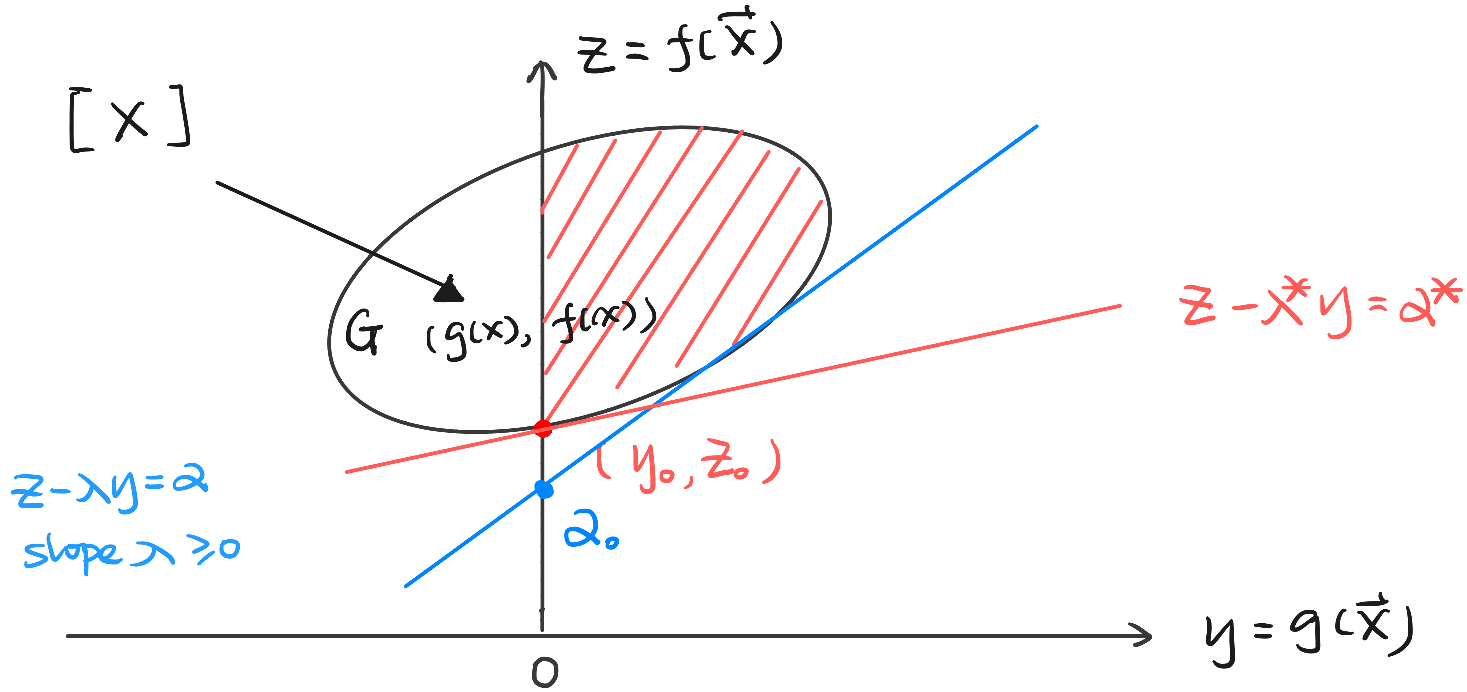
maximize $L^*(\lambda)$ subject to $\lambda \geq 0$



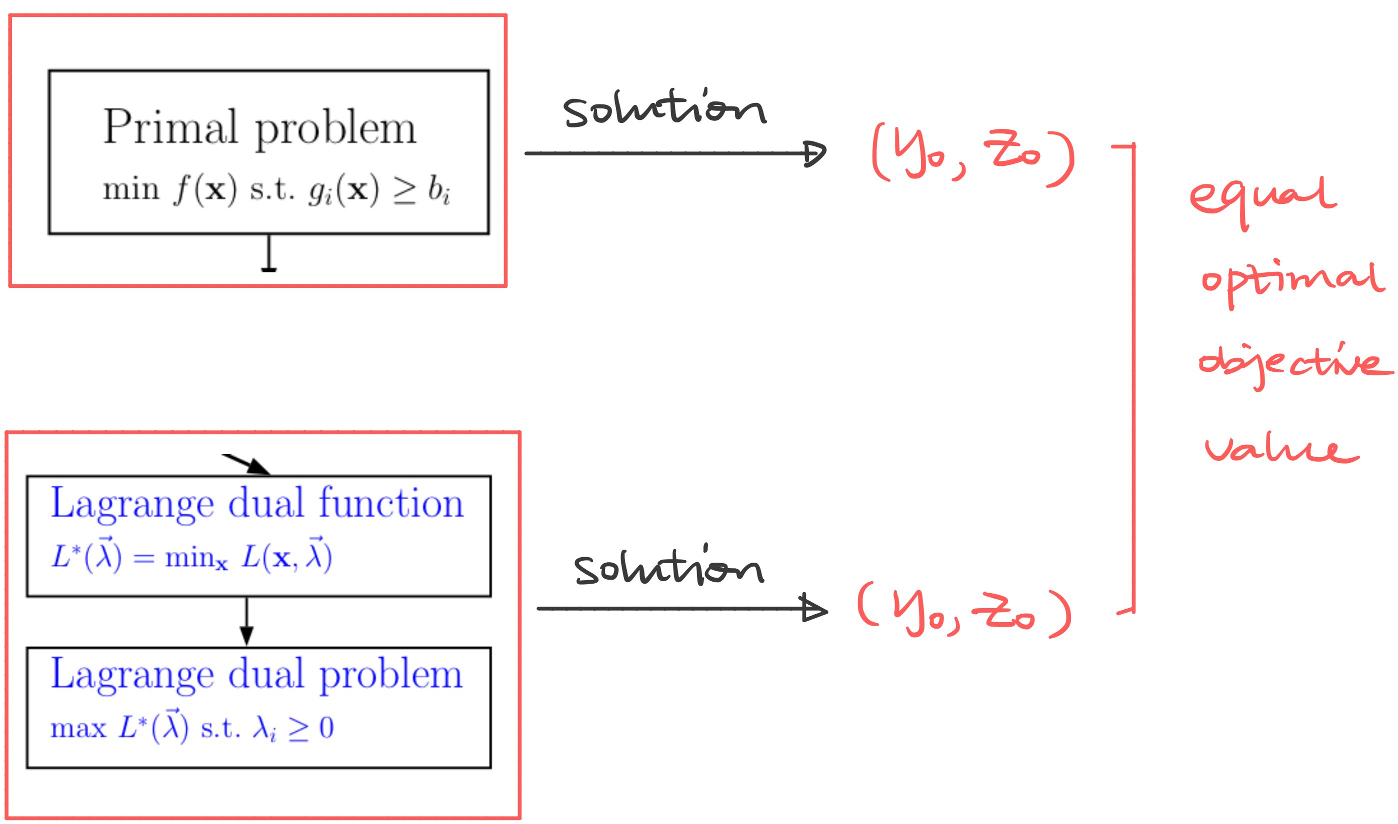
▼ Have to find the line with slope λ ($\lambda \geq 0$) to maximize the intercept α_0

▼ Such a line has slope λ^* and supports the set G at the point (y_0, z_0)

▼ The solution to the dual problem is λ^* and the optimal dual objective value is z_0



$z = f(\vec{x})$
[x]
G
$(g(x), f(x))$
$z - \lambda^* y = \alpha^*$
(y_0, z_0)
$z - \lambda y = \alpha$
slope $\lambda \geq 0$
α_0
$y = g(\vec{x})$
0



Primal problem
$\min f(\mathbf{x})$ s.t. $g_i(\mathbf{x}) \geq b_i$
solution
(y_0, z_0)
equal
optimal
objective
value
Lagrange dual function
$L^*(\vec{\lambda}) = \min_{\mathbf{x}} L(\mathbf{x}, \vec{\lambda})$
Lagrange dual problem
$\max L^*(\vec{\lambda})$ s.t. $\lambda_i \geq 0$
solution
(y_0, z_0)