Worksheet 9: Coordinate system

Example 0.61. Find the coordinate vector of $\mathbf{x} = [2, 5]^T \in \mathbb{R}^2$ relative to the basis given by the columns of $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Example 0.62. We have previously showed that the columns of the matrix form a basis for $\mathbb{R}^3$:

$$\mathbf{A} = \begin{bmatrix} 3 & -4 & -2 \\ 0 & 1 & 1 \\ -6 & 7 & 5 \end{bmatrix}$$

and for $\mathbf{b} = [1\ 0\ 2]^T \in \mathbb{R}^3$, we obtained that

$$\mathbf{b} = (-1)\mathbf{a}_1 + (-2)\mathbf{a}_2 + 2\mathbf{a}_3.$$

Therefore, the coordinates of $\mathbf{b}$ relative to the basis (columns of $\mathbf{A}$) are $[-1, -2, 2]^T$.

Example 0.63. Let $V = \{\text{all polynomials of degree at most } 2\}$. Then V is a vector space with basis $\mathcal{B} = \{1, t, t^2\}$. The coordinate mapping from V to $\mathbb{R}^3$ is

$$\mathbf{v} = c_0 + c_1 t + c_2 t^2 \quad \mapsto \quad [\mathbf{v}]_{\mathcal{B}} = [c_0, c_1, c_2]^T \in \mathbb{R}^3.$$

It can be shown that this is a 1-to-1 linear transformation (isomorphism) from V to $\mathbb{R}^3$ and the two vector spaces are identical algebraically.

Example 0.64. Let $\mathbf{v}_1 = [1, 1]^T$. Then $\mathcal{B} = \{\mathbf{v}_1\}$ is a basis for $H = \text{Span}\{\mathbf{v}_1\} \subset \mathbb{R}^2$. Determine if $\mathbf{x} = [5, 5]^T$ is in H, and if yes, find its coordinate vector relative to $\mathcal{B}$.

Example 0.65. Let $\mathbf{v}_1 = [1, 1, 0]^T, \mathbf{v}_2 = [1, 0, 1]^T$. Then $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for $H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\} \subset \mathbb{R}^3$. Determine if $\mathbf{x} = [3, 2, 1]^T$ is in H, and if yes, find its coordinate vector relative to $\mathcal{B}$.