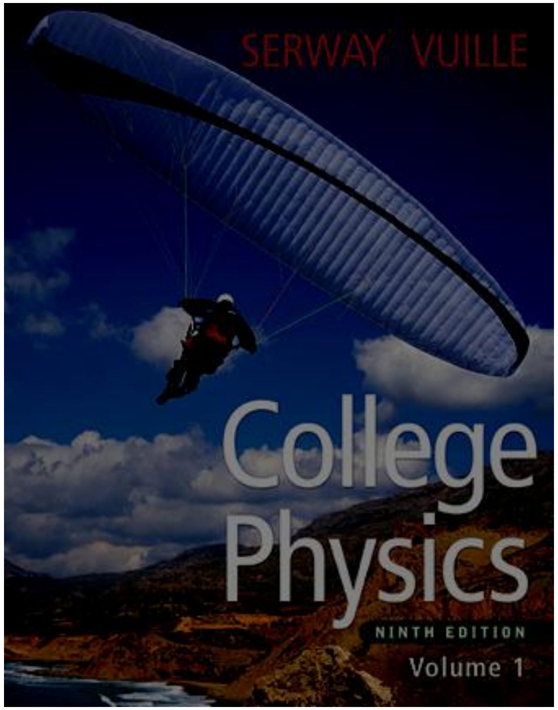

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Chapter 16

Electric Energy and Capacitance

Potential Energy

- The concept of potential energy is useful in the study of electricity.
- A potential energy function can be defined corresponding to the electric force.
- Electric potential can also be defined.
- The concept of potential relates to circuits.

Electric Potential Energy

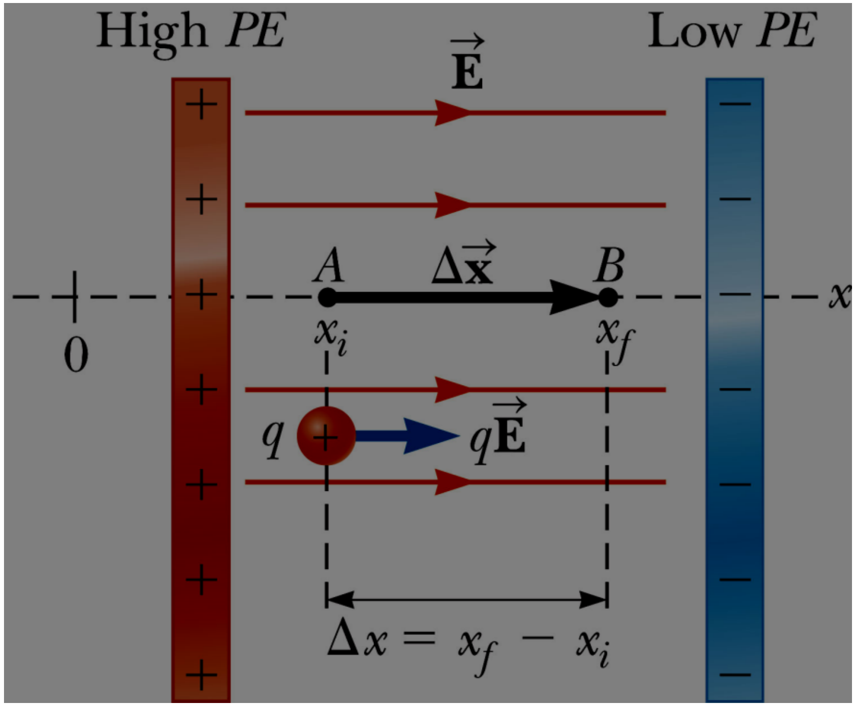
- The Coulomb force is a conservative force.
- It is possible to define an electrical potential energy function with this force.
- Work done by a conservative force is equal to the negative of the change in potential energy.

Work and Potential Energy

- There is a uniform field between the two plates.
- As the charge moves from A to B, work is done on it.
- $W_{AB} = F_x \Delta x = q E_x (x_f – x_i)$
- $\Delta PE = - W_{AB} = - q E_x$ ⍰x

–Only for a uniform field for a particle that undergoes a displacement along a given axis

- SI unit of energy: J



Potential Difference

- The electric potential difference *ΔV* between points A and B is defined as the change in the potential energy (final value minus initial value) of a charge q moved from A to B divided by the size of the charge.
  - $\Delta V = V_B - V_A = \Delta PE / q$
- Potential difference is *not* the same as potential energy.

Potential Difference, Cont.

- Another way to relate the energy and the potential difference: $\Delta PE = q\,\Delta V$
- Both electric potential energy and potential difference are *scalar* quantities.
- Units of potential difference
  - V = J/C
- A special case occurs when there is a *uniform electric field.*
  - ⍰V = -E_x ⍰x
    - Gives more information about units: N/C = V/m

Potential Energy Compared to Potential

- Electric potential is characteristic of the field only.
  - Independent of any test charge that may be placed in the field
- Electric potential energy is characteristic of the charge-field system.
  - Due to an interaction between the field and the charge placed in the field

Electric Potential and Charge Movements

- When released from rest, positive charges accelerate spontaneously from regions of high potential to low potential.
- When released from rest, negative charges will accelerated from regions of low potential toward region of high potential.
- Work must be done on a negative charges to make them go in the direction of lower electric potential.

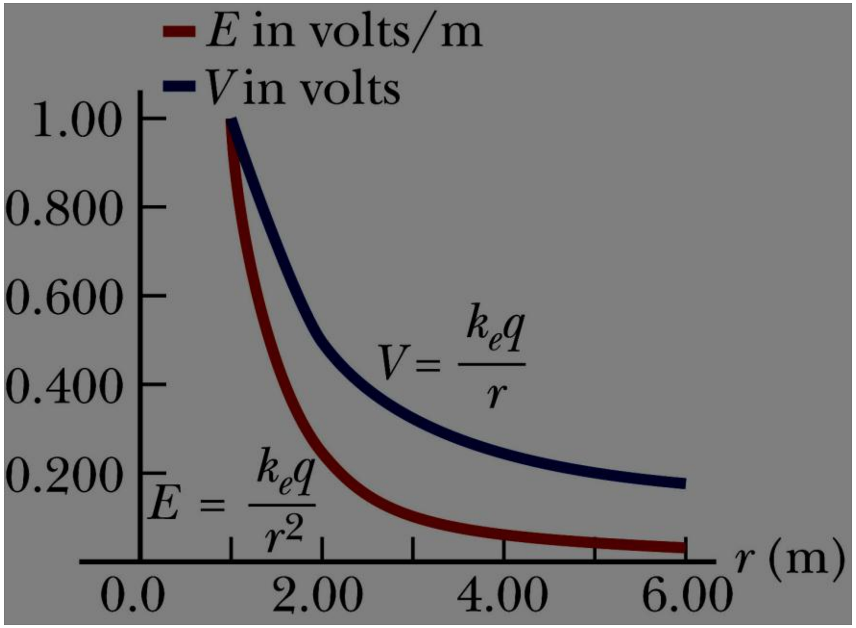
Electric Potential of a Point Charge

- The point of zero electric potential is taken to be at an infinite distance from the charge.
- The potential created by a point charge q at any distance r from the charge is

$$V = k_e \frac{q}{r}$$

Electric Field and Electric Potential Depend on Distance

- The electric field is proportional to $1/r^2$
- The electric potential is proportional to $1/r$



If the conservative force is exerted by electrostatic field E on point charge q, then the force is given by

$$\vec{F} = q\vec{E}$$

and if the point charge q undergoes a displacement $d\vec{\ell}$, the corresponding change in the electrostatic potential energy is given by

$$dU = -q\vec{E} \cdot d\vec{\ell} \qquad \text{23-1}$$

$$dV = \frac{dU}{q_0} = -\vec{E} \cdot d\vec{\ell} \qquad \text{23-2}a$$

DEFINITION—POTENTIAL DIFFERENCE

For a finite displacement from point a to point b, the change in potential is

$$\Delta V = V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E} \cdot d\vec{\ell} \qquad \text{23-2}b$$

The potential difference $V_b - V_a$ is the negative of the work per unit charge done by the electric field on a test charge when the test charge moves from point *a* to point *b* (along *any* path). During this calculation, the positions of any and all other charges remain fixed. (Recall that a test charge is a point charge whose magnitude is so small that it exerts only negligible forces on any and all other charges. For convenience, test charges are invariably considered to be positive charges.)

The function V is called the **electric potential;** it is often referred to as the **potential.** Like the electric field, the potential V is a function of position. Unlike the electric field, V is a scalar function, whereas $\vec{E}$ is a vector function. As with potential energy U, only *differences* in the potential V are physically significant. We are free to choose the potential to be zero at any convenient point, just as we are when dealing with potential energy. For convenience, the electric potential and the potential energy of a test charge are chosen to be zero at the same reference point. Under this restriction they are related by

$$U = q_0 V \qquad 23\text{-}3$$

(*a*) What is the electric potential at a distance $r_0 = 0.529 \times 10^{-10}$ m from a proton? This is the average distance between the proton and the electron in a hydrogen atom. (*b*) What is the electric potential energy of the electron and the proton at this separation?

$$V = \frac{kq}{r_0} = \frac{ke}{r_0} = \frac{(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C})}{0.529 \times 10^{-10}\ \mathrm{m}}$$

$$= 27.2\ \mathrm{N \cdot m/C} = \boxed{27.2\ \mathrm{V}}$$

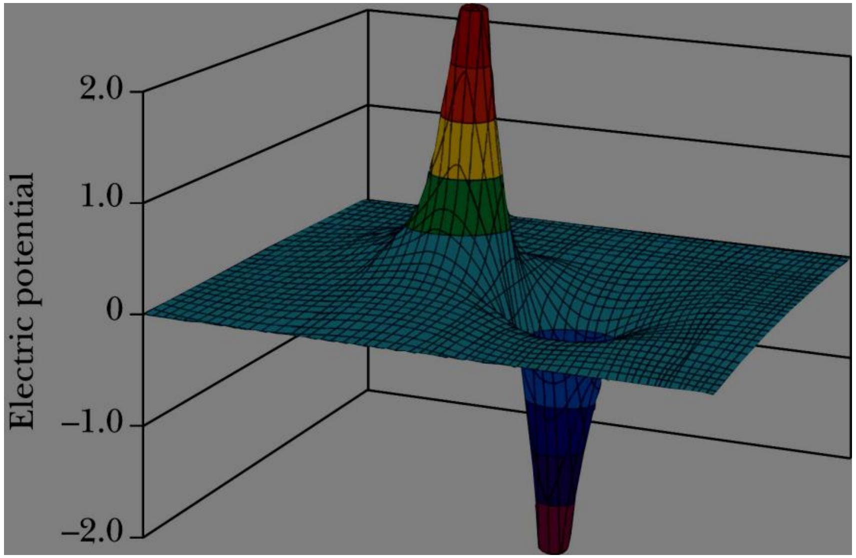
$$U = q'V = (-e)(27.2\ \mathrm{V}) = \boxed{-27.2\ \mathrm{eV}}$$

Electric Potential of Multiple Point Charges

- Superposition principle applies
- The total electric potential at some point P due to several point charges is the *algebraic* sum of the electric potentials due to the individual charges.
  - The algebraic sum is used because potentials are scalar quantities.

Dipole Example

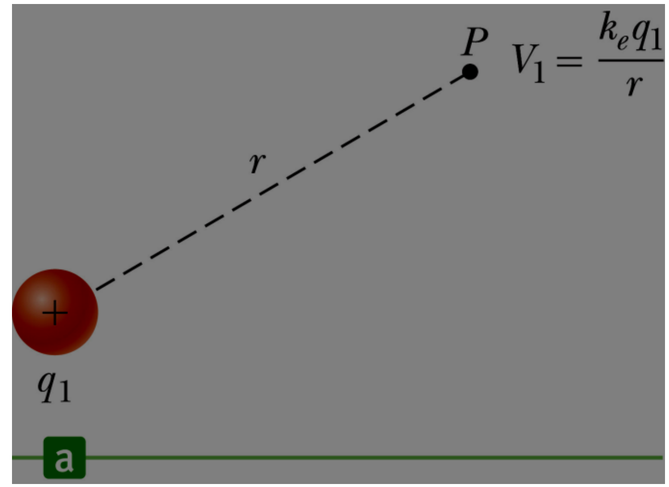
- Potential is plotted on the vertical axis.
  - In arbitrary units
- Two charges have equal magnitudes and opposite charges.
- Example of superposition

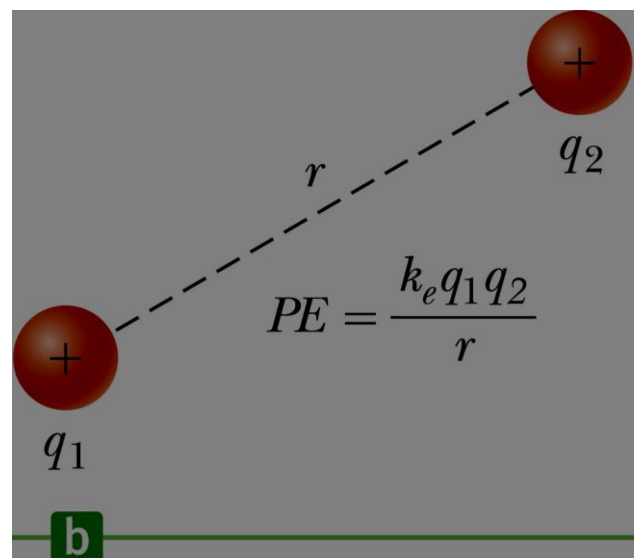


Electrical Potential Energy of Two Charges

- V_1 is the electric potential due to q_1 at some point P
- The work required to bring q_2 from infinity to P without acceleration is q_2V_1
- This work is equal to the potential energy of the two particle system

$$PE = q_2V_1 = k_e\frac{q_1q_2}{r}$$

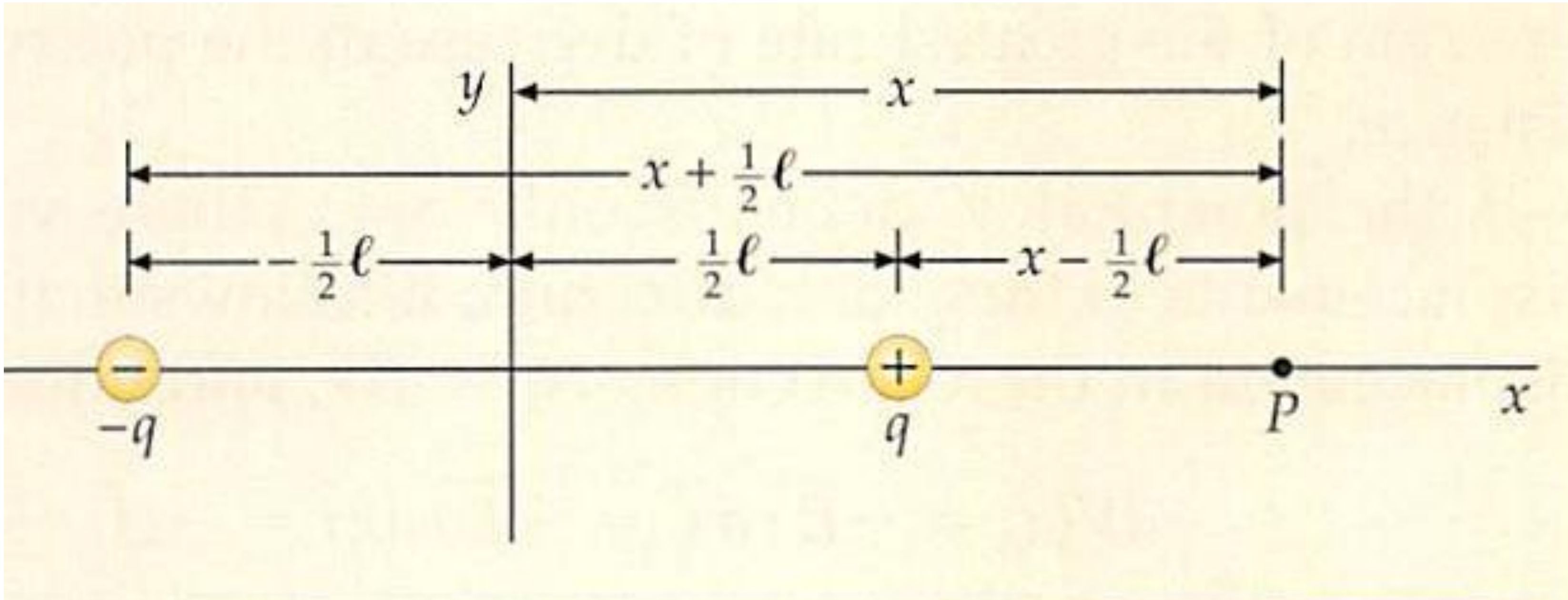




Notes About Electric Potential Energy of Two Charges

- If the charges have the *same* sign, PE is positive.
  - Positive work must be done to force the two charges near one another.
  - The like charges would repel.
- If the charges have *opposite* signs, PE is negative.
  - The force would be attractive.
  - Work must be done to hold back the unlike charges from accelerating as they are brought close together.

An electric dipole consists of a positive point charge $+q$ on the x axis at $x = +\ell/2$ and a negative point charge $-q$ on the x axis at $x = -\ell/2$. Find the potential on the x axis for $x \gg +\ell/2$ in terms of the dipole moment $\vec{p} = q\ell\hat{i}$.



Sketch the x axis and place the two charges on it. For $x > \ell/2$, the distance from the field point P to the positive charge is $x - \frac{1}{2}\ell$ and the distance from the field point to the negative charge is $x + \frac{1}{2}\ell$

For $x > \ell/2$, the potential due to the two charges is

$$V = \frac{kq}{x - (\ell/2)} + \frac{k(-q)}{x + (\ell/2)}$$

$$= \frac{kq\ell}{x^2 - (\ell^2/4)} \qquad x > \frac{\ell}{2}$$

The magnitude of $\vec{p}$ is $p = q\ell$. For $x \gg \ell/2$, we can neglect $\ell^2/4$ compared with x^2 in the denominator.

$$V \approx \boxed{\frac{kq\ell}{x^2} = \frac{kp}{x^2} \qquad x \gg \ell}$$

Problem Solving with Electric Potential (Point Charges)

- Draw a diagram of all charges.
  - Note the point of interest.
- Calculate the distance from each charge to the point of interest.
- Use the basic equation $V = k_e q/r$
  - Include the sign
  - The potential is positive if the charge is positive and negative if the charge is negative.

Problem Solving with Electric Potential, Cont.

- Use the superposition principle when you have multiple charges.
  - Take the algebraic sum
- Remember that potential is a scalar quantity.
  - So no components to worry about

Potentials and Charged Conductors

- Since $W = -q(V_B - V_A)$, no net work is required to move a charge between two points that are at the same electric potential.
  - $W = 0$ when $V_A = V_B$
- All points on the surface of a charged conductor in electrostatic equilibrium are at the same potential.
- Therefore, the electric potential is constant everywhere on the surface of a charged conductor in electrostatic equilibrium.

The Electron Volt

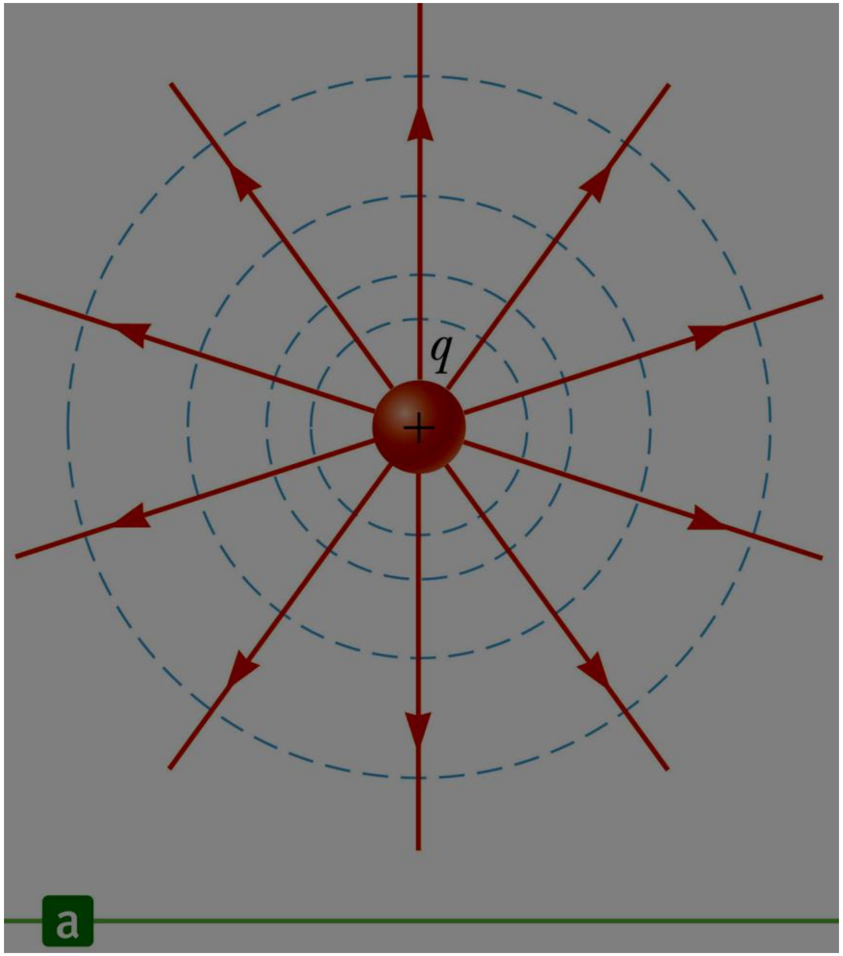
- The electron volt (eV) is defined as the kinetic energy that an electron gains when accelerated through a potential difference of 1 V.
  - Electrons in normal atoms have energies of 10’s of eV.
  - Excited electrons have energies of 1000’s of eV.
  - High energy gamma rays have energies of millions of eV.
- 1 eV = 1.6×10^{-19} J

Equipotential Surfaces

- An *equipotential surface* is a surface on which all points are at the same potential.
  - No work is required to move a charge at a constant speed on an equipotential surface.
  - The electric field at every point on an equipotential surface is perpendicular to the surface.

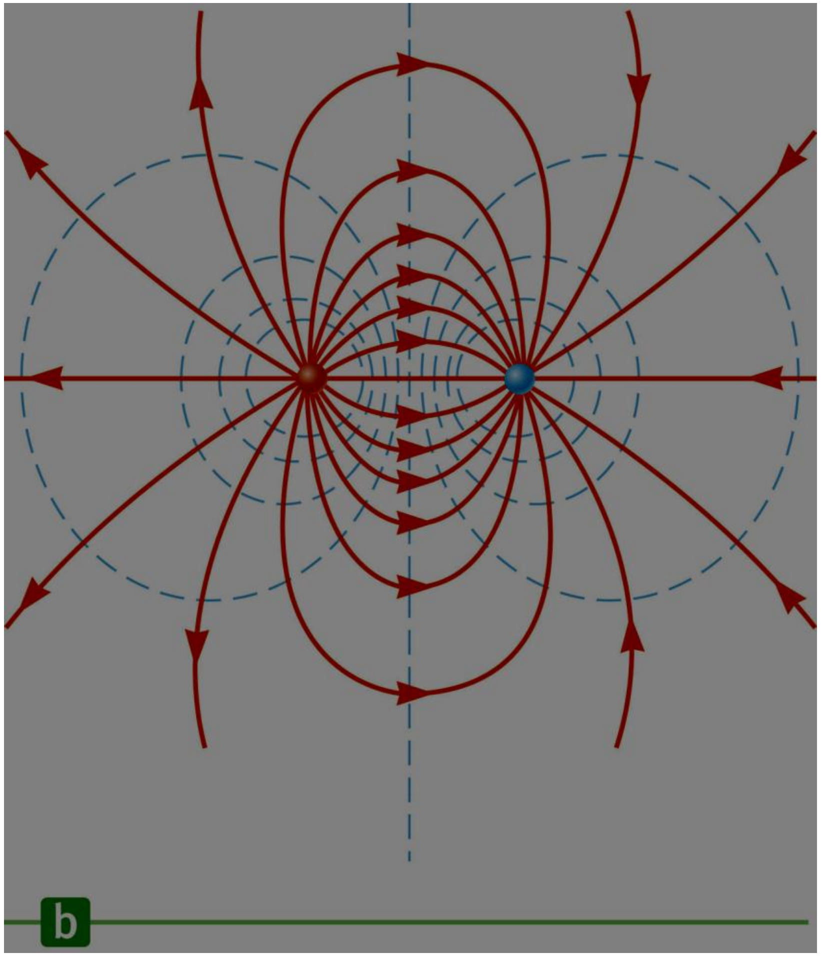
Equipotentials and Electric Fields Lines – Positive Charge

- The equipotentials for a point charge are a family of spheres centered on the point charge.
  - In blue
- The field lines are perpendicular to the electric potential at all points.
  - In orange



Equipotentials and Electric Fields Lines – Dipole

- Equipotential lines are shown in blue.
- Electric field lines are shown in orange.
- The field lines are perpendicular to the equipotential lines at all points.



Capacitance

- A capacitor is a device used in a variety of electric circuits.
- The *capacitance*, C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor (plate) to the magnitude of the potential difference between the conductors (plates).

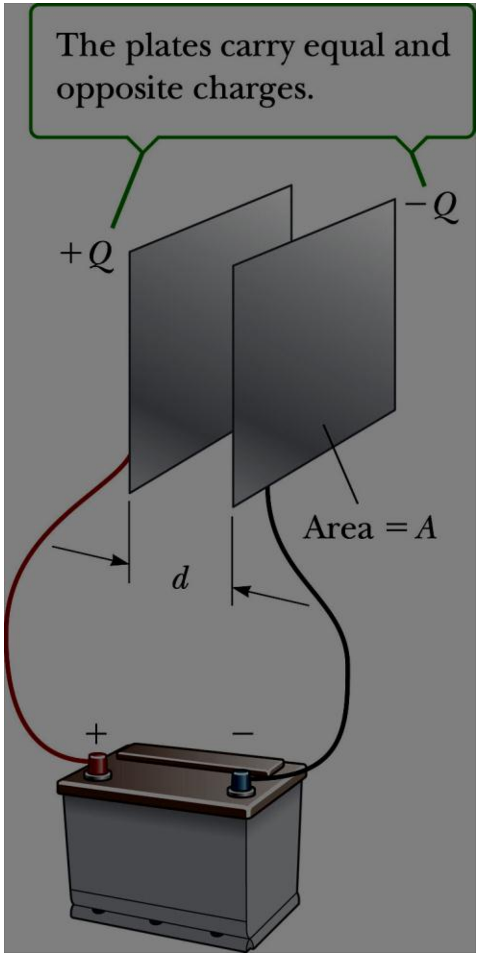
Capacitance, Cont.

- $$C \equiv \frac{Q}{\Delta V}$$
- Units: Farad (F)
  - 1 F = 1 C / V
  - A Farad is very large
    - Often will see µF or pF

- ΔV is the potential difference across a circuit element or device.
- V represents the actual potential due to a given charge at a given location.

Parallel-Plate Capacitor, Example

- The capacitor consists of two parallel plates.
- Each has area A.
- They are separated by a distance d.
- The plates carry equal and opposite charges.
- When connected to the battery, charge is pulled off one plate and transferred to the other plate.
- The transfer stops when $\Delta V_{cap} = \Delta V_{battery}$



The capacitance C of a capacitor is the ratio of the magnitude of the charge on either conductor (plate) to the magnitude of the potential difference between the conductors (plates):

$$C \equiv \frac{Q}{\Delta V} \qquad [16.8]$$

SI unit: farad (F) = coulomb per volt (C/V)

The quantities Q and ΔV are always taken to be positive when used in Equation 16.8. For example, if a 3.0-μF capacitor is connected to a 12-V battery, the magnitude of the charge on each plate of the capacitor is

$$Q = C\Delta V = (3.0 \times 10^{-6}\ \text{F})(12\ \text{V}) = 36\ \mu\text{C}$$

From Equation 16.8, we see that a large capacitance is needed to store a large amount of charge for a given applied voltage. The farad is a very large unit of capacitance. In practice, most typical capacitors have capacitances ranging from microfarads ($1\ \mu\text{F} = 1 \times 10^{-6}\ \text{F}$) to picofarads ($1\ \text{pF} = 1 \times 10^{-12}\ \text{F}$).

$$C = \frac{Q}{\Delta V} = \frac{\sigma A}{Ed} = \frac{\sigma A}{(\sigma/\epsilon_0)d}$$

Canceling the charge per unit area, σ, yields

$$C = \epsilon_0 \frac{A}{d} \qquad [16.9]$$

where A is the area of one of the plates, d is the distance between the plates, and ϵ_0 is the permittivity of free space.

PROBLEM A parallel-plate capacitor has an area $A = 2.00 \times 10^{-4}\ m^2$ and a plate separation $d = 1.00 \times 10^{-3}$ m. (a) Fin its capacitance. (b) How much charge is on the positive plate if the capacitor is connected to a 3.00-V battery? Calcula (c) the charge density on the positive plate, assuming the density is uniform, and (d) the magnitude of the electric fie between the plates.

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})\left(\frac{2.00 \times 10^{-4}\,\mathrm{m^2}}{1.00 \times 10^{-3}\,\mathrm{m}}\right)$$

$$C = \boxed{1.77 \times 10^{-12}\,\mathrm{F} = 1.77\,\mathrm{pF}}$$

$$C = \frac{Q}{\Delta V} \quad \rightarrow \quad Q = C\,\Delta V = (1.77 \times 10^{-12}\,\mathrm{F})(3.00\,\mathrm{V})$$

$$= \boxed{5.31 \times 10^{-12}\,\mathrm{C}}$$

$$\sigma = \frac{Q}{A} = \frac{5.31 \times 10^{-12}\,\mathrm{C}}{2.00 \times 10^{-4}\,\mathrm{m^2}} = \boxed{2.66 \times 10^{-8}\,\mathrm{C/m^2}}$$

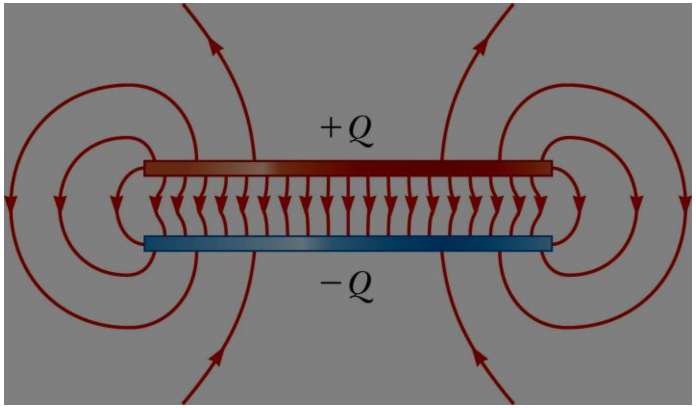
$$E = \frac{\Delta V}{d} = \frac{3.00\,\mathrm{V}}{1.00 \times 10^{-3}\,\mathrm{m}} = \boxed{3.00 \times 10^{3}\,\mathrm{V/m}}$$

Parallel-Plate Capacitor

- The capacitance of a device depends on the geometric arrangement of the conductors.
- For a parallel-plate capacitor whose plates are separated by air:

$$C = \varepsilon_o \frac{A}{d}$$

Electric Field in a Parallel-Plate Capacitor



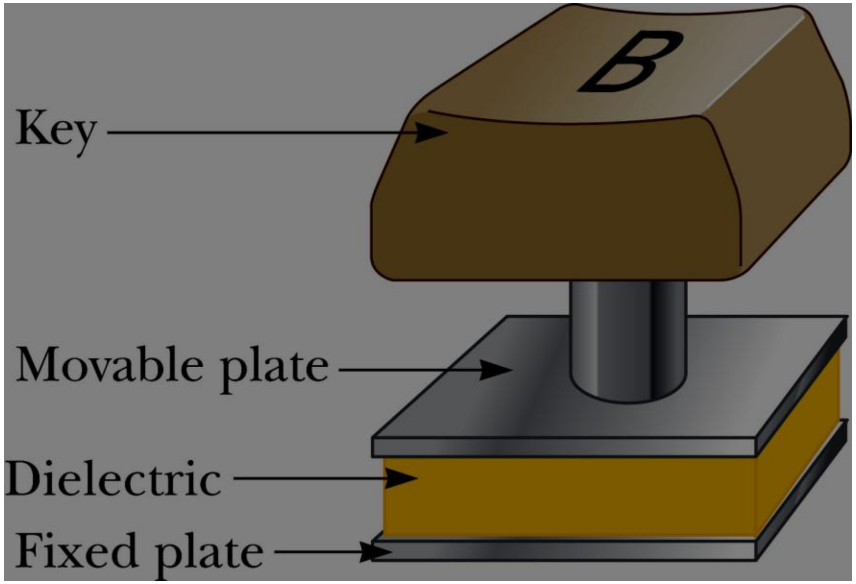
- The electric field between the plates is uniform.
  - Near the center
  - Nonuniform near the edges
- The field may be taken as constant throughout the region between the plates.

Application – Camera Flash

- The flash attachment on a camera uses a capacitor.
  - A battery is used to charge the capacitor.
  - The energy stored in the capacitor is released when the button is pushed to take a picture.
  - The charge is delivered very quickly, illuminating the subject when more light is needed.

Application – Computers

- Computers use capacitors in many ways.
  - –Some keyboards use capacitors at the bases of the keys.
  - –When the key is pressed, the capacitor spacing decreases and the capacitance increases.
  - –The key is recognized by the change in capacitance.

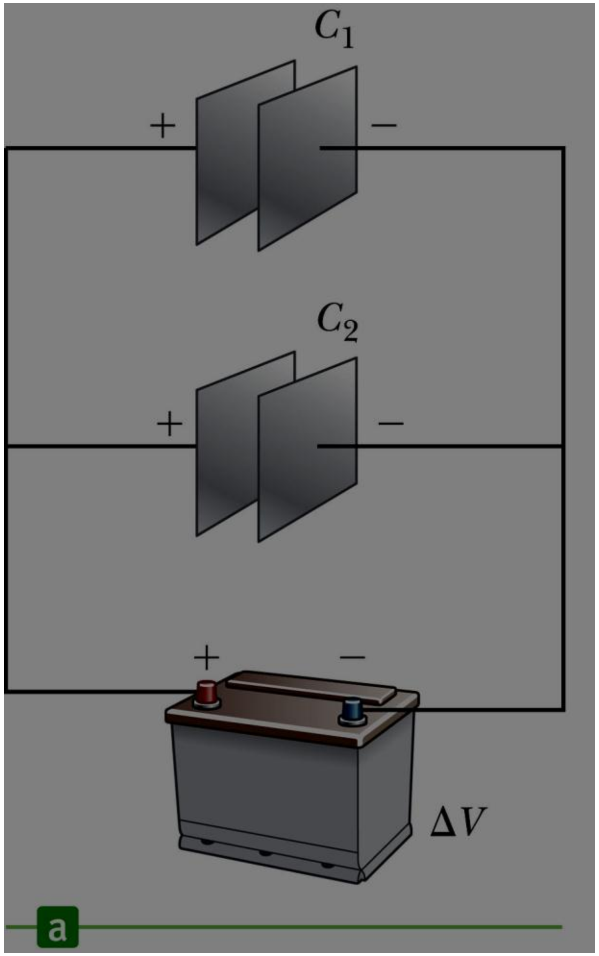


Capacitors in Circuits

- A *circuit* is a collection of objects usually containing a source of electrical energy (such as a battery) connected to elements that convert electrical energy to other forms.
- A *circuit diagram* can be used to show the path of the real circuit.

Capacitors in Parallel

- When connected in parallel, both have the same potential difference, V, across them.

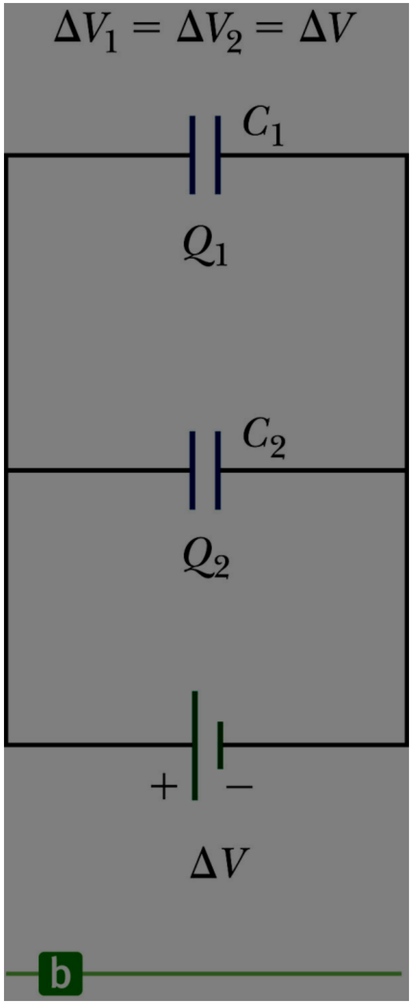


Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.
- The flow of charges ceases when the voltage across the capacitors equals that of the battery.
- The capacitors reach their maximum charge when the flow of charge ceases.

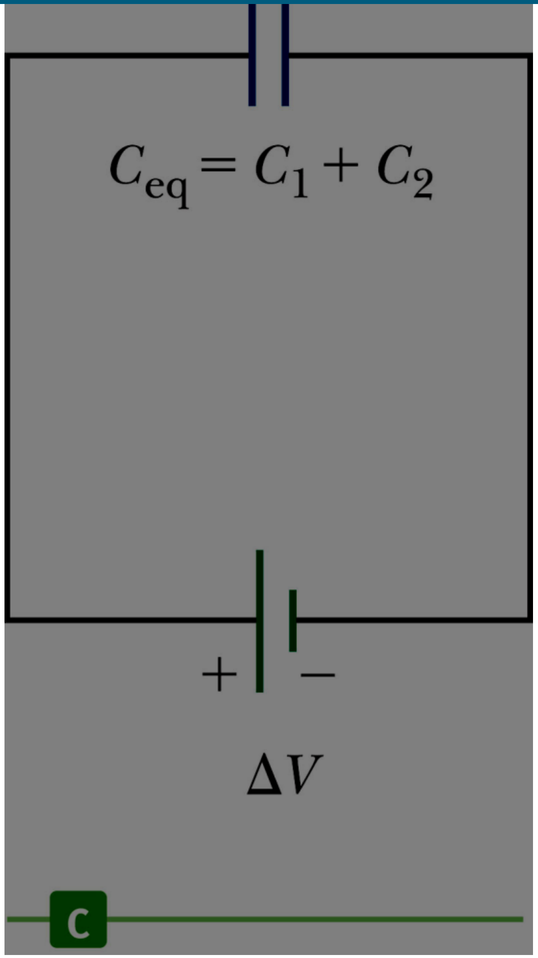
Capacitors in Parallel

- The potential difference across the capacitors is the same.
  - And each is equal to the voltage of the battery
- The total charge, Q, is equal to the sum of the charges on the capacitors.
  - $Q = Q_1 + Q_2$



More About Capacitors in Parallel

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}
  - The equivalent capacitor must have exactly the same external effect on the circuit as the original capacitors.



$$Q = Q_1 + Q_2 \qquad [16.10]$$

We can replace these two capacitors with one equivalent capacitor having a capacitance of C_{eq}. This equivalent capacitor must have exactly the same external effect on the circuit as the original two, so it must store Q units of charge and have the same potential difference across it. The respective charges on each capacitor are

$$Q_1 = C_1\,\Delta V \quad \text{and} \quad Q_2 = C_2\,\Delta V$$

The charge on the equivalent capacitor is

$$Q = C_{eq}\,\Delta V$$

Substituting these relationships into Equation 16.10 gives

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

or

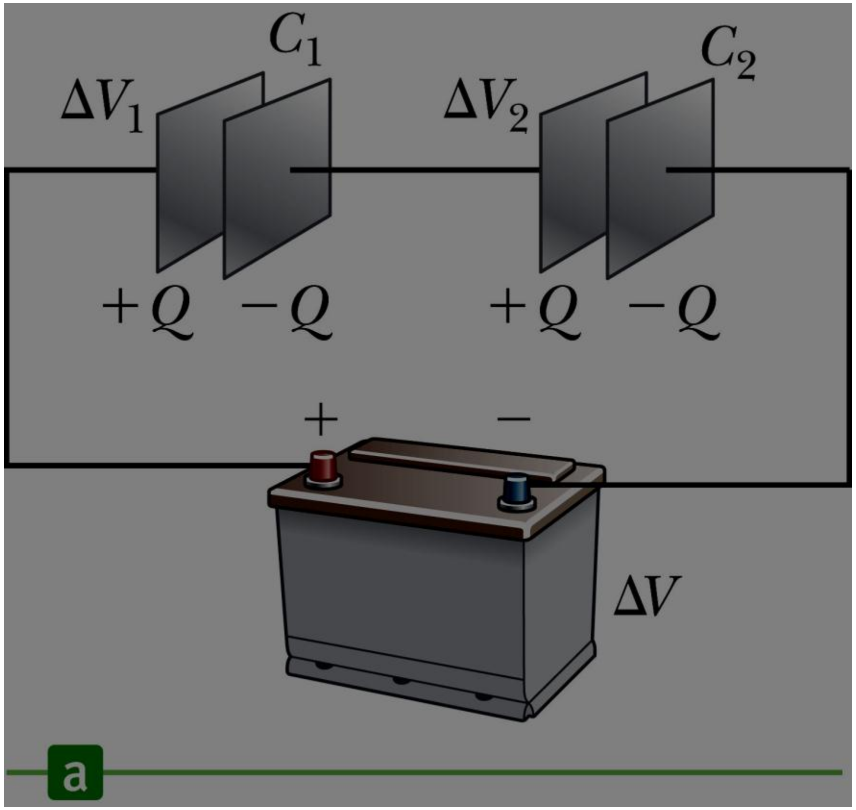
$$C_{eq} = C_1 + C_2 \quad \begin{pmatrix}\text{parallel}\\ \text{combination}\end{pmatrix} \qquad [16.11]$$

Capacitors in Parallel, Final

- $C_{eq} = C_1 + C_2 + \ldots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.

Capacitors in Series

- When in series, the capacitors are connected end-to-end.
- The magnitude of the charge must be the same on all the plates.

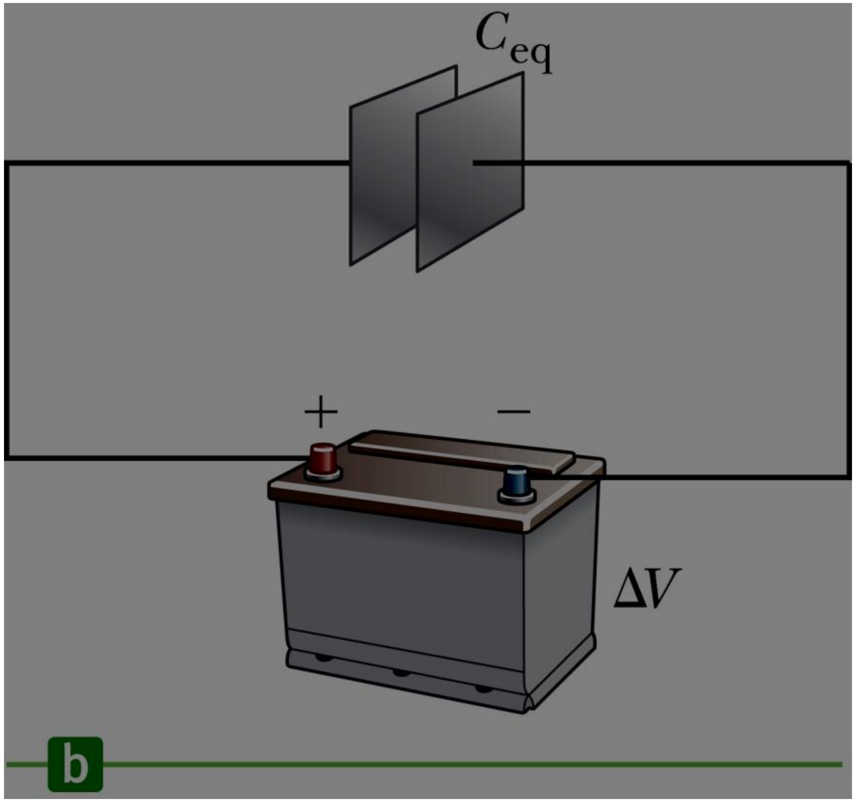


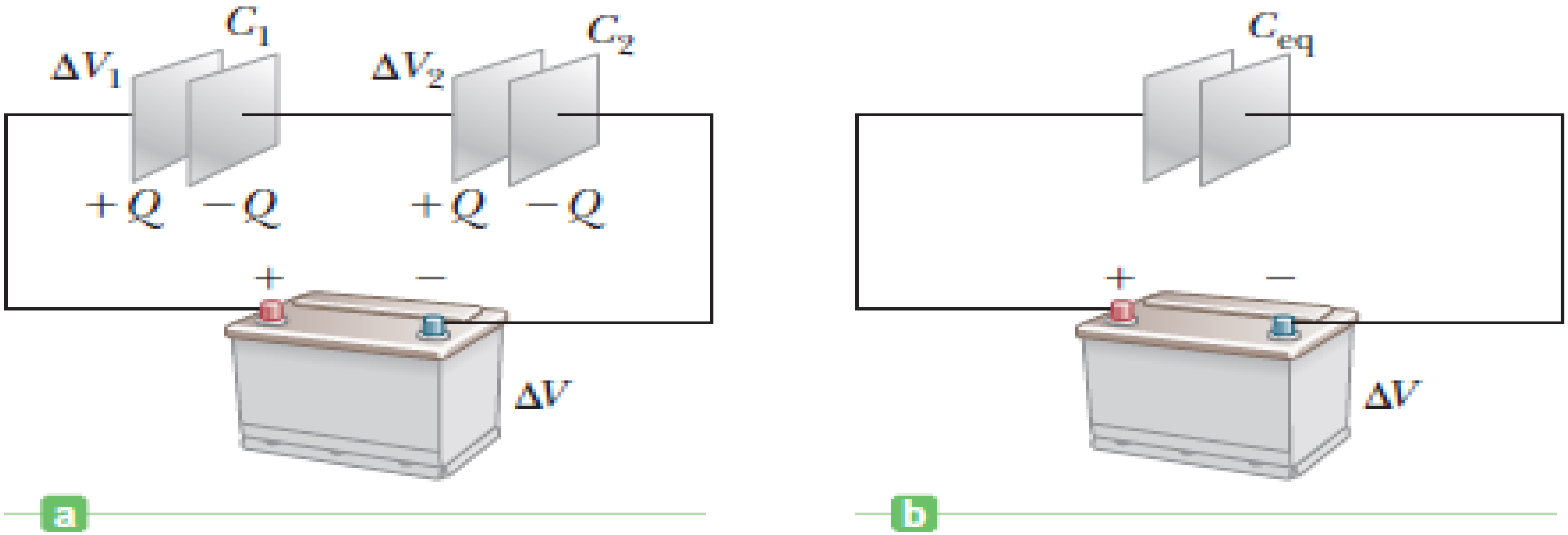
Capacitors in Series

- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.
- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.
- All of the right plates gain charges of –Q and all the left plates have charges of +Q.

More About Capacitors in Series

- An equivalent capacitor can be found that performs the same function as the series combination.
- The potential differences add up to the battery voltage.





$$\Delta V = \frac{Q}{C_{eq}}$$

where ΔV is the potential difference between the terminals of the battery and C_{eq} is the equivalent capacitance. Because $Q = C\Delta V$ can be applied to each capacitor, the potential differences across them are given by

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

From Active Figure 16.20a, we see that

$$\Delta V = \Delta V_1 + \Delta V_2 \qquad [16.13]$$

where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2 (a consequence of the conservation of energy).

The potential difference across any number of capacitors (or other circuit elements) in series equals the sum of the potential differences across the individual capacitors. Substituting these expressions into Equation 16.13 and noting that $\Delta V = Q/C_{eq}$, we have

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling Q, we arrive at the following relationship:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \begin{pmatrix}\text{series} \\ \text{combination}\end{pmatrix} \qquad [16.14]$$

Capacitors in Series, Final

- $$\Delta V = V_1 + V_2$$
  $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination.

Problem-Solving Strategy

- Be careful with the choice of units.
- **Combine** capacitors following the formulas.

–When two or more unequal capacitors are connected *in series*, they carry the same charge, but the potential differences across them are not the same.

- The capacitances add as reciprocals and the equivalent capacitance is always less than the smallest individual capacitor.

–When two or more capacitors are connected *in parallel*, the potential differences across them are the same.

- The charge on each capacitor is proportional to its capacitance.
- The capacitors add directly to give the equivalent capacitance.

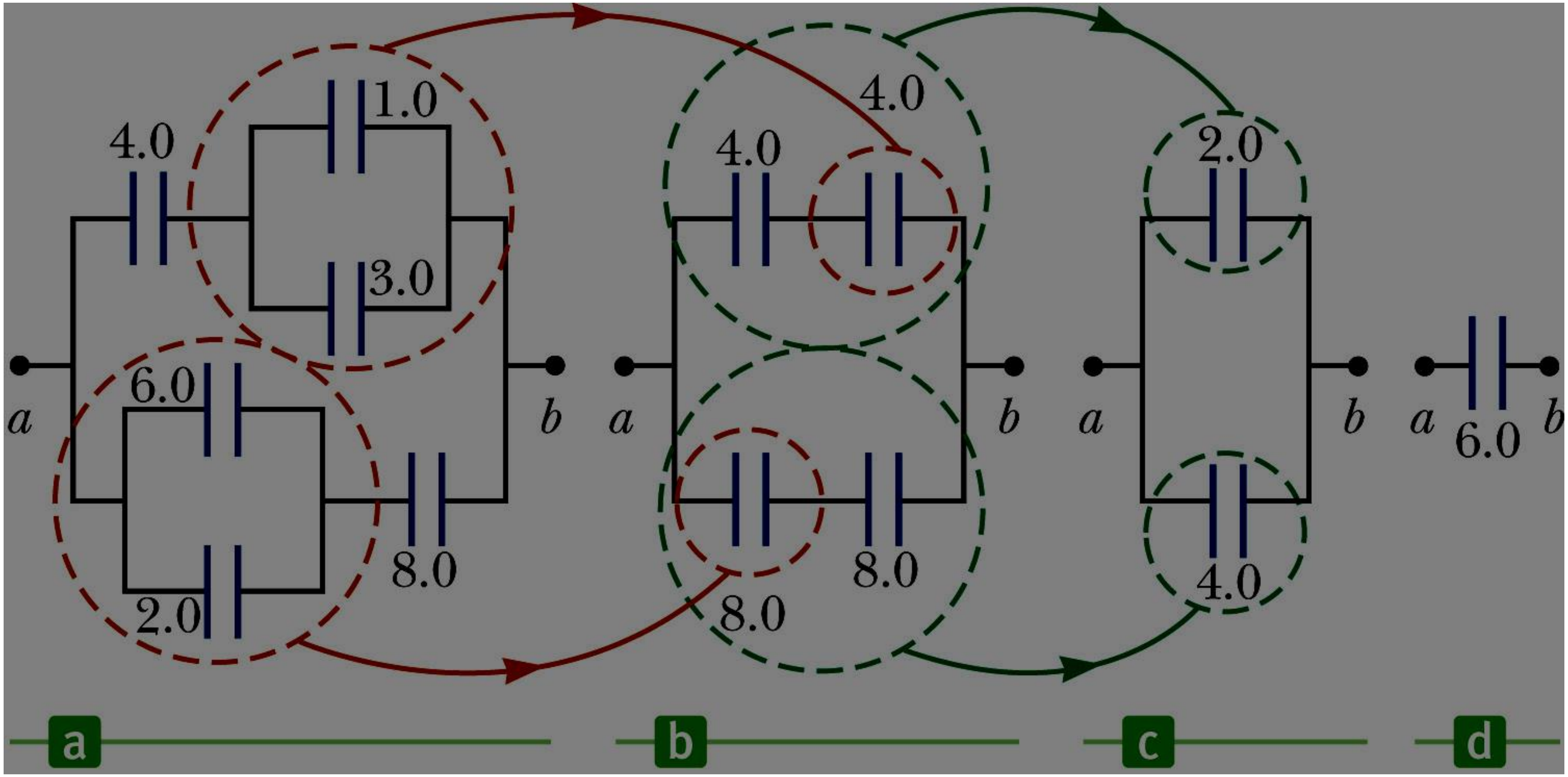
Problem-Solving Strategy, Final

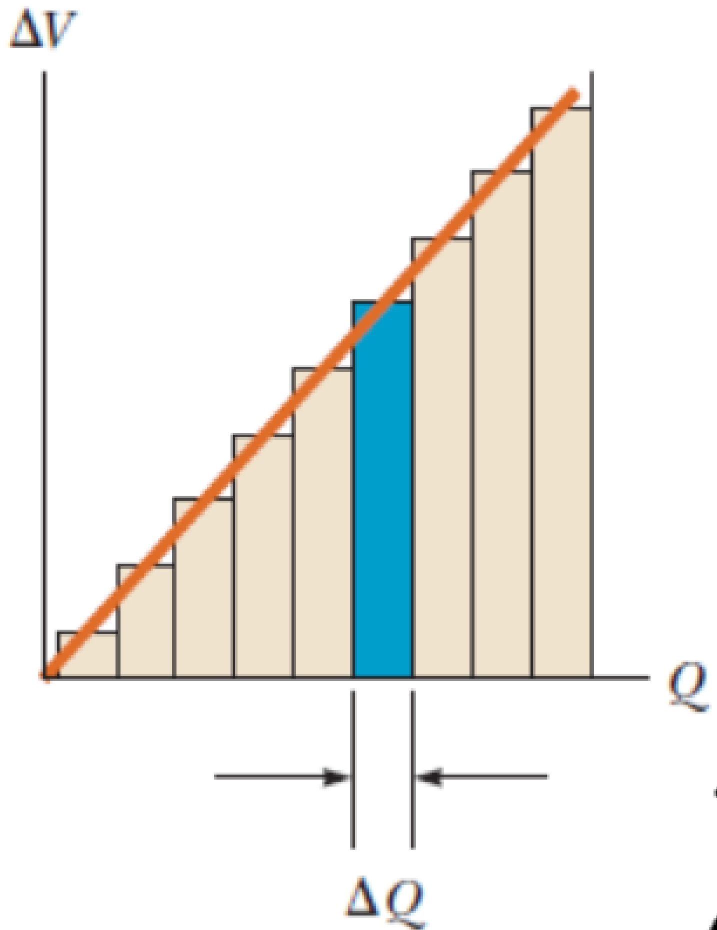
- **Redraw** the circuit after every combination.
- **Repeat** the process until there is only one single equivalent capacitor.
  - A complicated circuit can often be reduced to one equivalent capacitor.
    - Replace capacitors in series or parallel with their equivalent.
    - Redraw the circuit and continue.
- To find the charge on, or the potential difference across, one of the capacitors, start with your final equivalent capacitor and work back through the circuit reductions.

Problem-Solving Strategy, Equation Summary

- Use the following equations when working through the circuit diagrams:
  - Capacitance equation: $C = Q / \Delta V$
  - Capacitors in parallel: $C_{eq} = C_1 + C_2 + \ldots$
  - Capacitors in parallel all have the same voltage differences as does the equivalent capacitance.
  - Capacitors in series: $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$
  - Capacitors in series all have the same charge, Q, as does their equivalent capacitance.

Circuit Reduction Example





$C = Q/\Delta V$

∴ ΔV vs. Q plot has a slope of $1/C$

Adding a charge ΔQ when potential is ΔV is $\Delta W = \Delta Q \cdot \Delta V$. ∴ W = Area of Δ

$= \frac{1}{2} Q \Delta V$

Energy Stored in a Capacitor

- Energy stored = ½ Q ΔV
- From the definition of capacitance, this can be rewritten in different forms.

$$\text{Energy} = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

As previously stated, W is also the energy stored in the capacitor. From the definition of capacitance, we have $Q = C\,\Delta V$; hence, we can express the energy stored three different ways:

$$\text{Energy stored} = \tfrac{1}{2}Q\Delta V = \tfrac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C} \qquad [16.17]$$

For example, the amount of energy stored in a 5.0-μF capacitor when it is connected across a 120-V battery is

$$\text{Energy stored} = \tfrac{1}{2}C(\Delta V)^2 = \tfrac{1}{2}(5.0 \times 10^{-6}\ \text{F})(120\ \text{V})^2 = 3.6 \times 10^{-2}\ \text{J}$$

In practice, there is a limit to the maximum energy (or charge) that can be stored in a capacitor. At some point, the Coulomb forces between the charges on the plates become so strong that electrons jump across the gap, discharging the capacitor. For this reason, capacitors are usually labeled with a maximum operating voltage.

(This physical fact can actually be exploited to yield a circuit with a regularly blinking light).

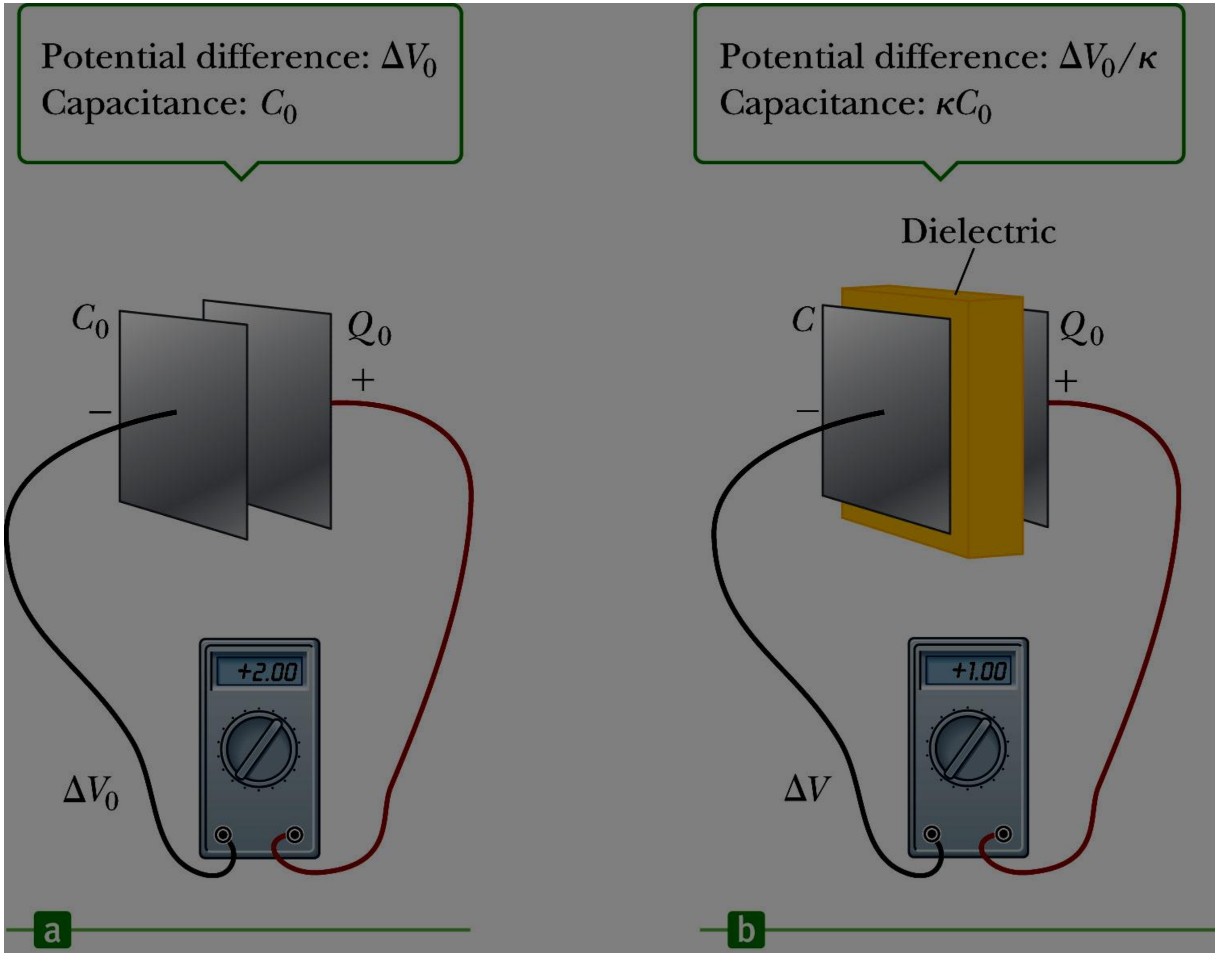
Application

- Defibrillators
  - When fibrillation occurs, the heart produces a rapid, irregular pattern of beats.
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern.
- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse.

Capacitors with Dielectrics

- A *dielectric* is an insulating material that, when placed between the plates of a capacitor, increases the capacitance.
  - Dielectrics include rubber, plastic, or waxed paper.
- $C = \kappa C_o = \kappa \varepsilon_o (A/d)$
  - The capacitance is multiplied by the factor κ when the dielectric completely fills the region between the plates.
  - κ is called the ***dielectric constant.***

Capacitors with Dielectrics

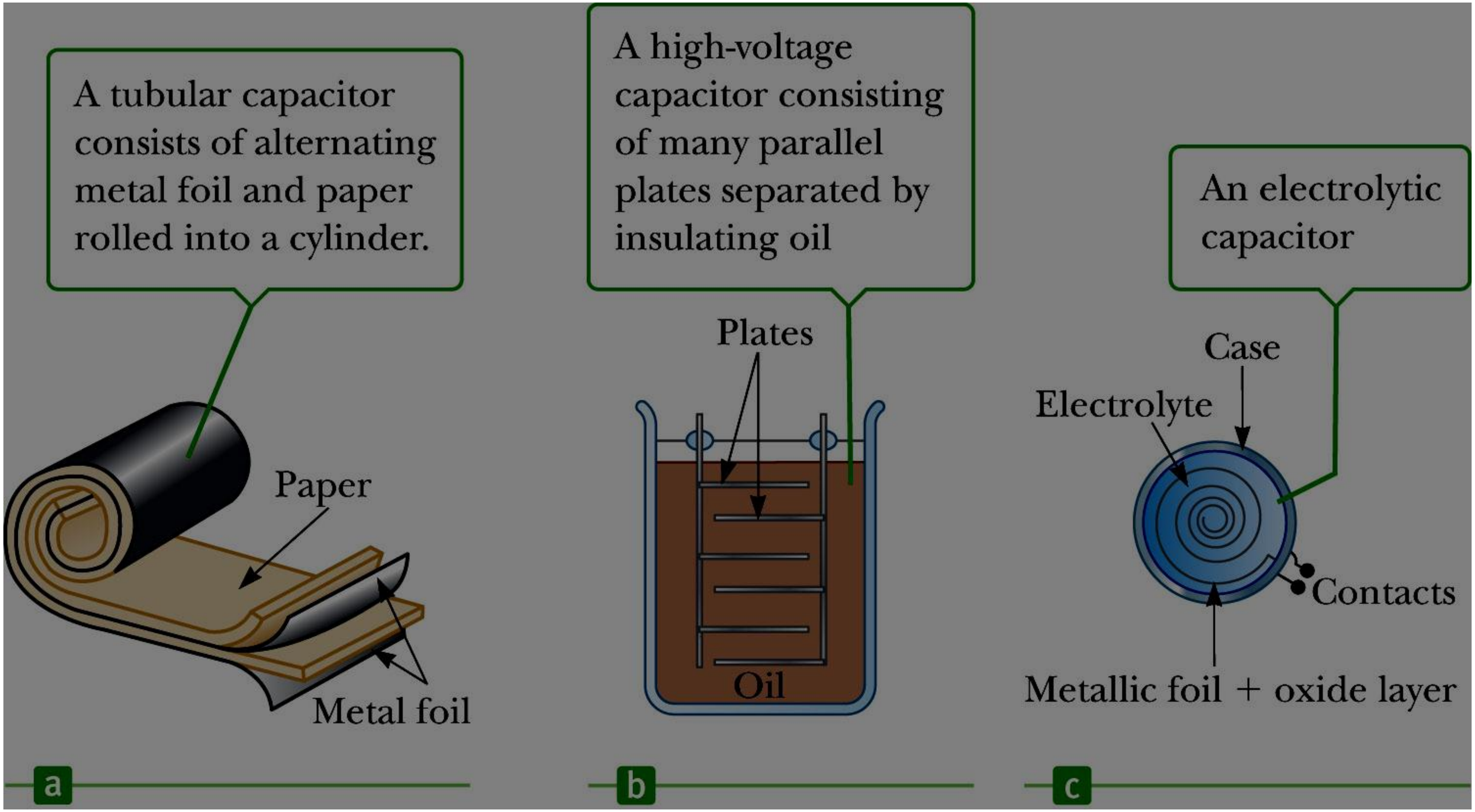


Section 16.10

Dielectric Strength

- For any given plate separation, there is a maximum electric field that can be produced in the dielectric before it breaks down and begins to conduct.
- This maximum electric field is called the **dielectric strength.**

Commercial Capacitor Designs



An Atomic Description of Dielectrics

- Polarization occurs when there is a separation between the average positions of its negative charge and its positive charge.
- In a capacitor, the dielectric becomes polarized because it is in an electric field that exists between the plates.
- The field produces an *induced polarization* in the dielectric material.

More Atomic Description

- The presence of the positive charge on the dielectric effectively reduces some of the negative charge on the metal
- This allows more negative charge on the plates for a given applied voltage
- The capacitance increases