4.54

$$E_1 = \frac{V_1}{d_1}$$

$$D_{n1} = D_{n2}$$

$$\varepsilon_1 \varepsilon_o E_1 = \varepsilon_2 \varepsilon_o E_2$$

$$\varepsilon_1 \left(\frac{V_1}{d_1} \right) = \varepsilon_2 \left(\frac{V_2}{d_2} \right) = \frac{\varepsilon_2}{d_2} (V - V_1)$$

$$\left(\frac{\varepsilon_1}{d_1} + \frac{\varepsilon_2}{d_2} \right) V_1 = \frac{\varepsilon_2}{d_2} V$$

$$V_1 = \frac{\frac{\varepsilon_2}{d_2}}{\frac{\varepsilon_1}{d_1} + \frac{\varepsilon_2}{d_2}} V = \frac{\varepsilon_2 d_1}{\varepsilon_1 d_2 + \varepsilon_2 d_1} V$$

$$E_1 = \frac{V_1}{d_1} = \frac{\varepsilon_2 V}{\varepsilon_1 d_2 + \varepsilon_2 d_1}$$

$$E_2 = \frac{\varepsilon_1 V}{\varepsilon_1 d_2 + \varepsilon_2 d_1}$$

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \left(\frac{\varepsilon_1 A}{d_1} \right) \left(\frac{\varepsilon_2 d_1}{\varepsilon_1 d_2 + \varepsilon_2 d_1} V \right)^2 = \frac{A}{2} \frac{\varepsilon_1 \varepsilon_2^2 d_1 V^2}{(\varepsilon_1 d_2 + \varepsilon_2 d_1)^2}$$

$$U = U_1 + U_2 = \frac{A}{2} \frac{\varepsilon_1 \varepsilon_2 V^2}{(\varepsilon_1 d_2 + \varepsilon_2 d_1)^2} (\varepsilon_2 d_1 + \varepsilon_1 d_2) = \frac{A}{2} \frac{\varepsilon_1 \varepsilon_2 V^2}{(\varepsilon_1 d_2 + \varepsilon_2 d_1)}$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A} = \frac{\varepsilon_2 d_1 + \varepsilon_1 d_2}{\varepsilon_1 \varepsilon_2 A} = \frac{1}{C}$$

$$U = \frac{1}{2} C V^2$$