Electrodynamics

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Static Fields (stationary charges, steady current)

$$\left\{\begin{array}{l} \oint \vec{E}\cdot d\vec{a} = \dfrac{Q_t}{\varepsilon_o} \\ \oint \vec{E}\cdot d\vec{\ell} = 0 \\ \oint \vec{B}\cdot d\vec{a} = 0 \\ \oint \vec{B}\cdot d\vec{\ell} = \mu_o I_t \end{array}\right\}$$

Gauss's Law

Conservative

No magnetic charge

Ampere's Law

$$\left\{\begin{array}{l} \nabla\cdot\vec{D} = \rho_f \\ \nabla\times\vec{E} = 0 \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{H} = \vec{J}_f \end{array}\right\}$$

linear, homogeneous, isotropic

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_o\varepsilon_r\vec{E} = \varepsilon_o\vec{E} + \vec{P}$$

$$\vec{B} = \mu\vec{H} = \mu_o\mu_r\vec{H} = \mu_o(\vec{H} + \vec{M})$$

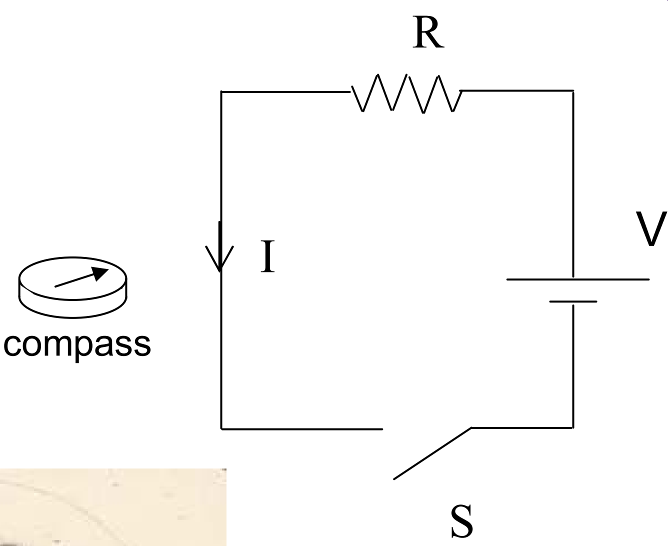
$$\nabla\cdot\vec{E} = \frac{\rho_t}{\varepsilon_o}$$

$$\nabla\cdot\vec{P} = -\rho_b$$

$$\rho_t = \rho_f + \rho_b = \frac{\rho_f}{\varepsilon_r} \le \rho_f$$

Orsted's Discovery

4/21/1820
Danmark



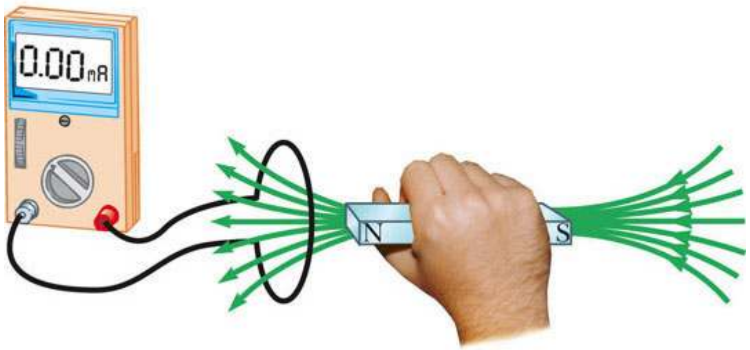
Hans Christian Ørsted
(1777–1851) Danmark
cgs unit for B

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_t$$

Ampere's Law, 9/18/1820,
after he learned about Orsted's
discovery on 9/11/1820 !!

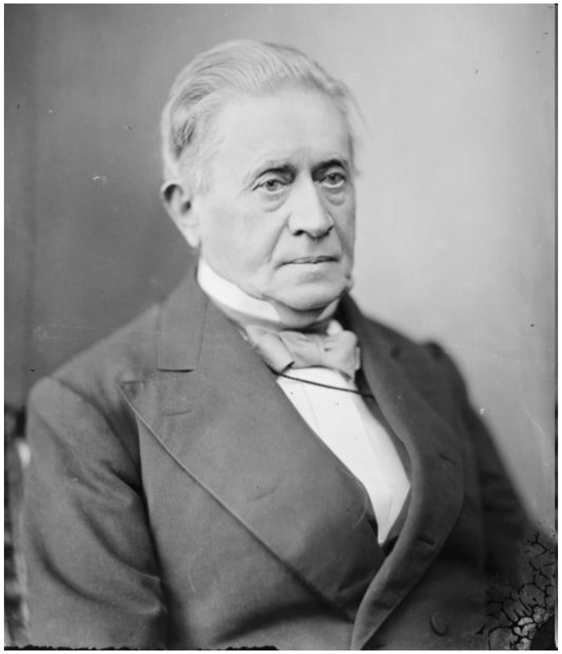
André-Marie Ampère
(1775–1836) France
SI unit for current

Faraday's Experiment



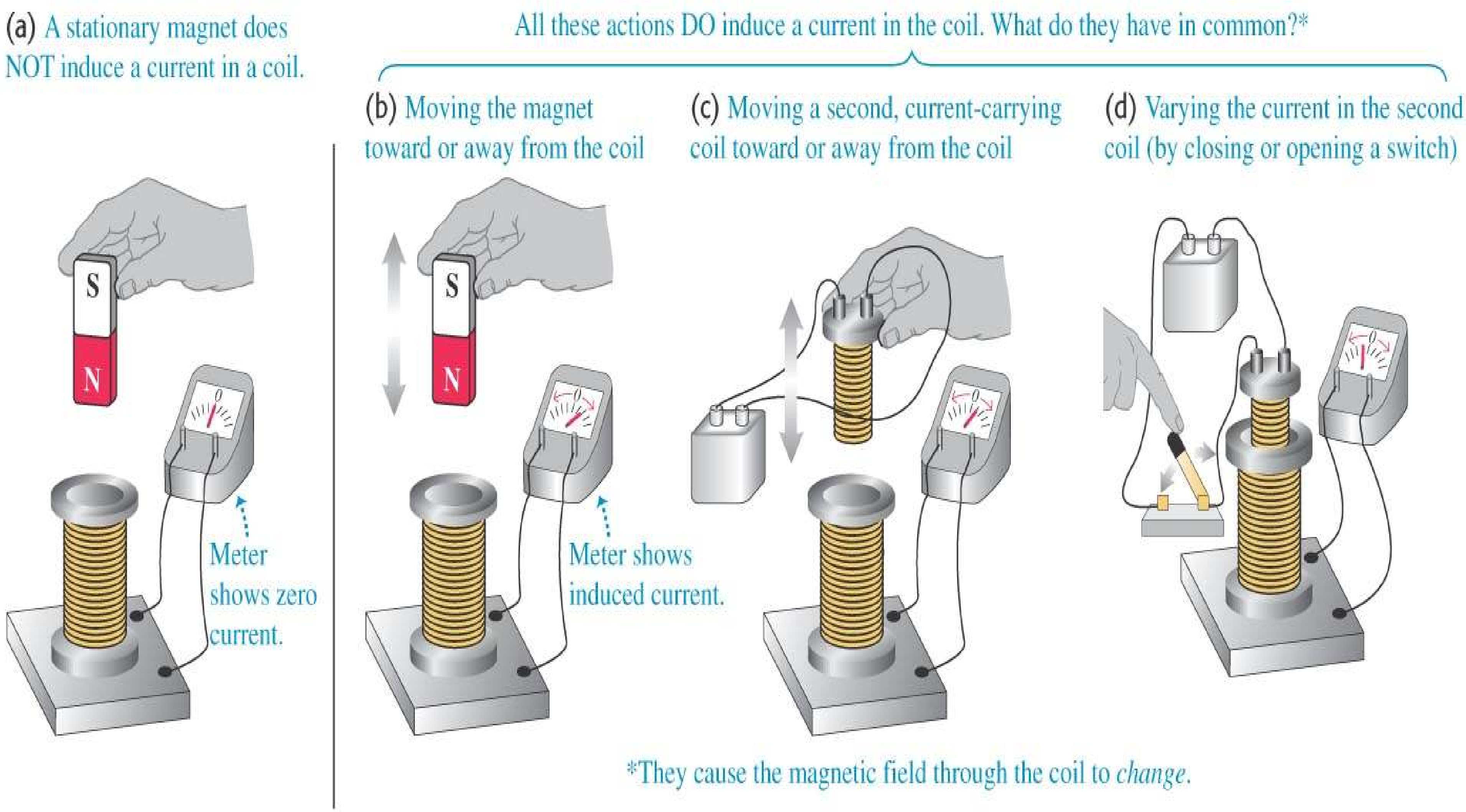
With stationary magnet,
no current induced (1831)

Michael Faraday
(1791–1867) England
SI unit for capacitance



Joseph Henry
(1797–1878) USA
SI unit for inductance

Magnetic induction



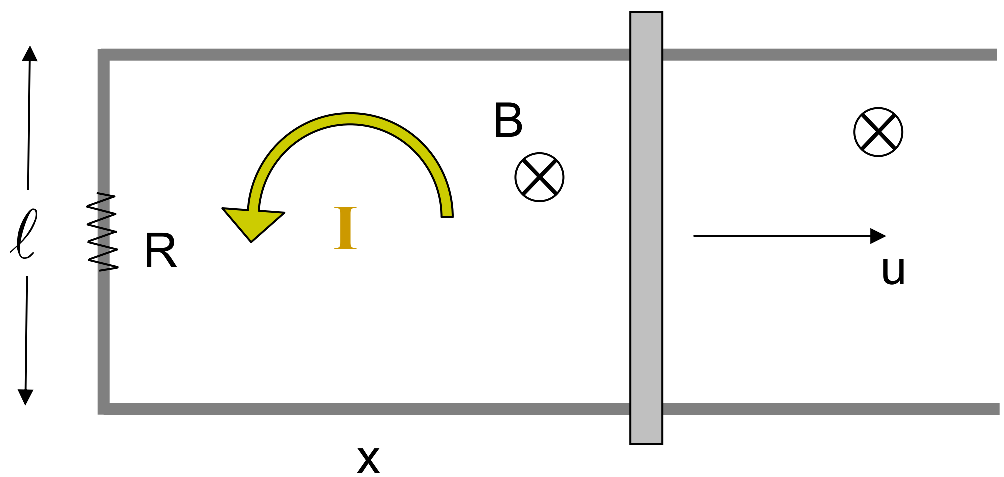
Faraday's Law

$$V_{emf} \equiv \oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi}{dt}$$

oppose the "change" of

$$\Phi = \int \vec{B}\cdot d\vec{a}$$

magnetic flux



$$\Phi = \int_0^x B\ell dx = B\ell x$$

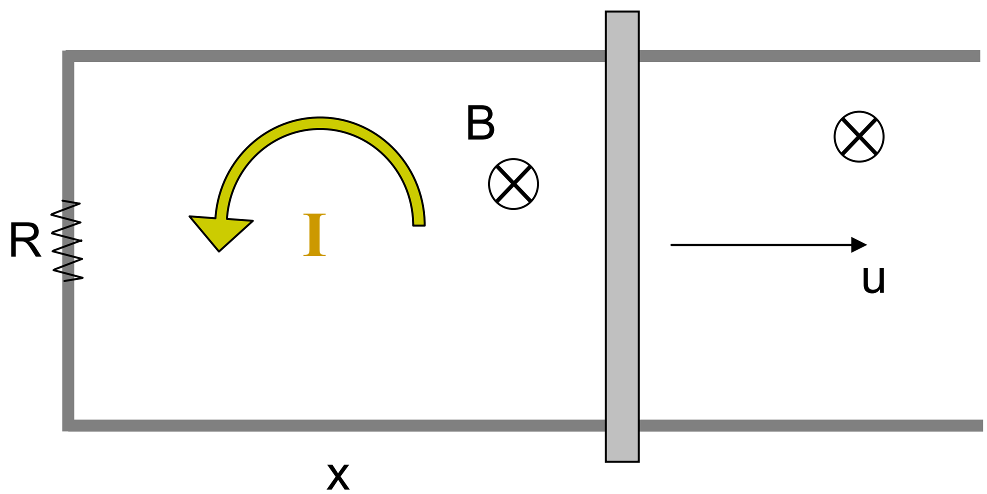
$$\frac{d\Phi}{dt} = B\ell\frac{dx}{dt} = B\ell u = -V_{emf}$$

$$I = \left|\frac{V_{emf}}{R}\right| = \frac{B\ell u}{R}$$

direction given by Lenz's Law

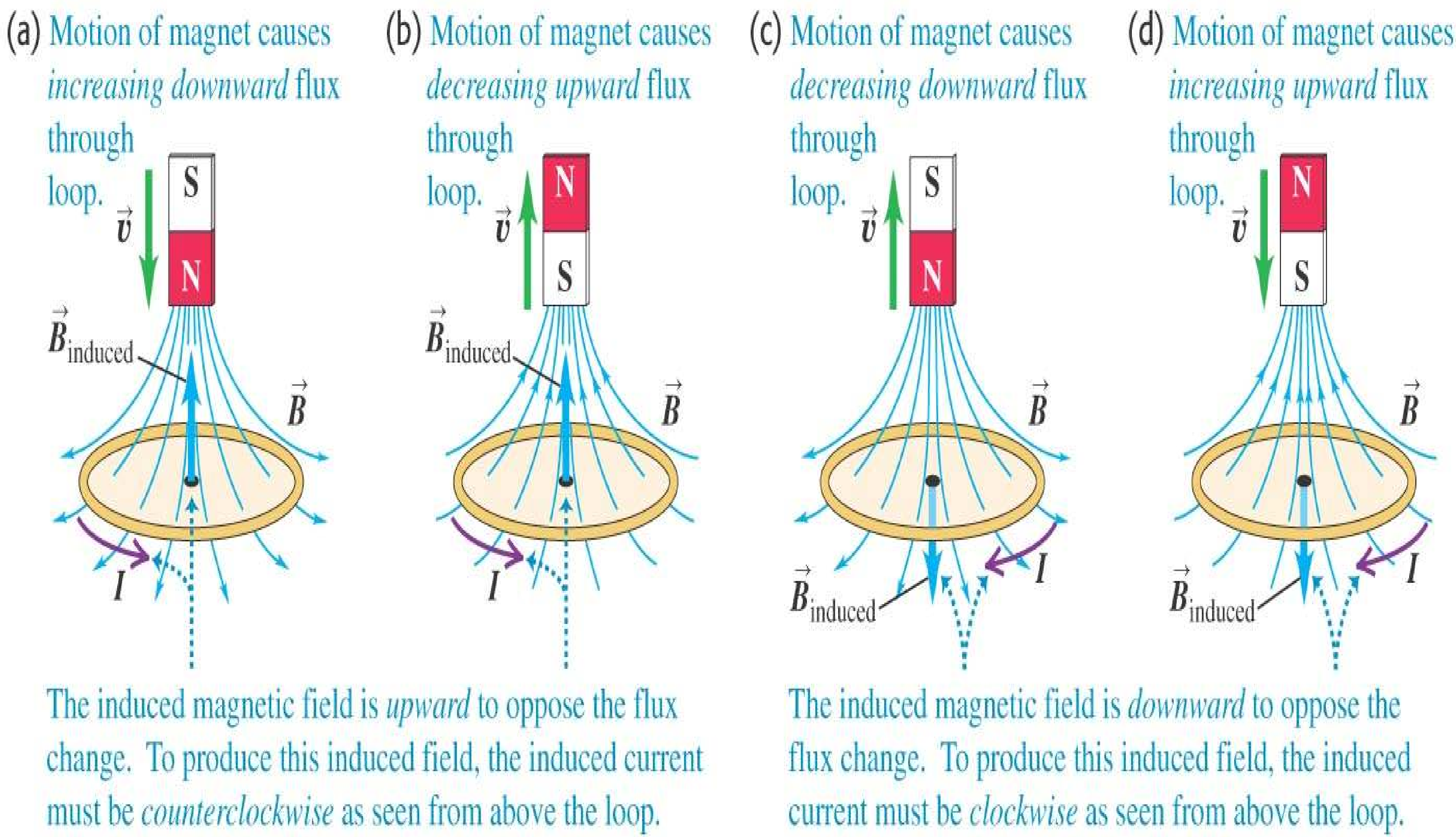
Lenz's Law (1834)

"Back emf" to oppose the "change" of magnetic flux

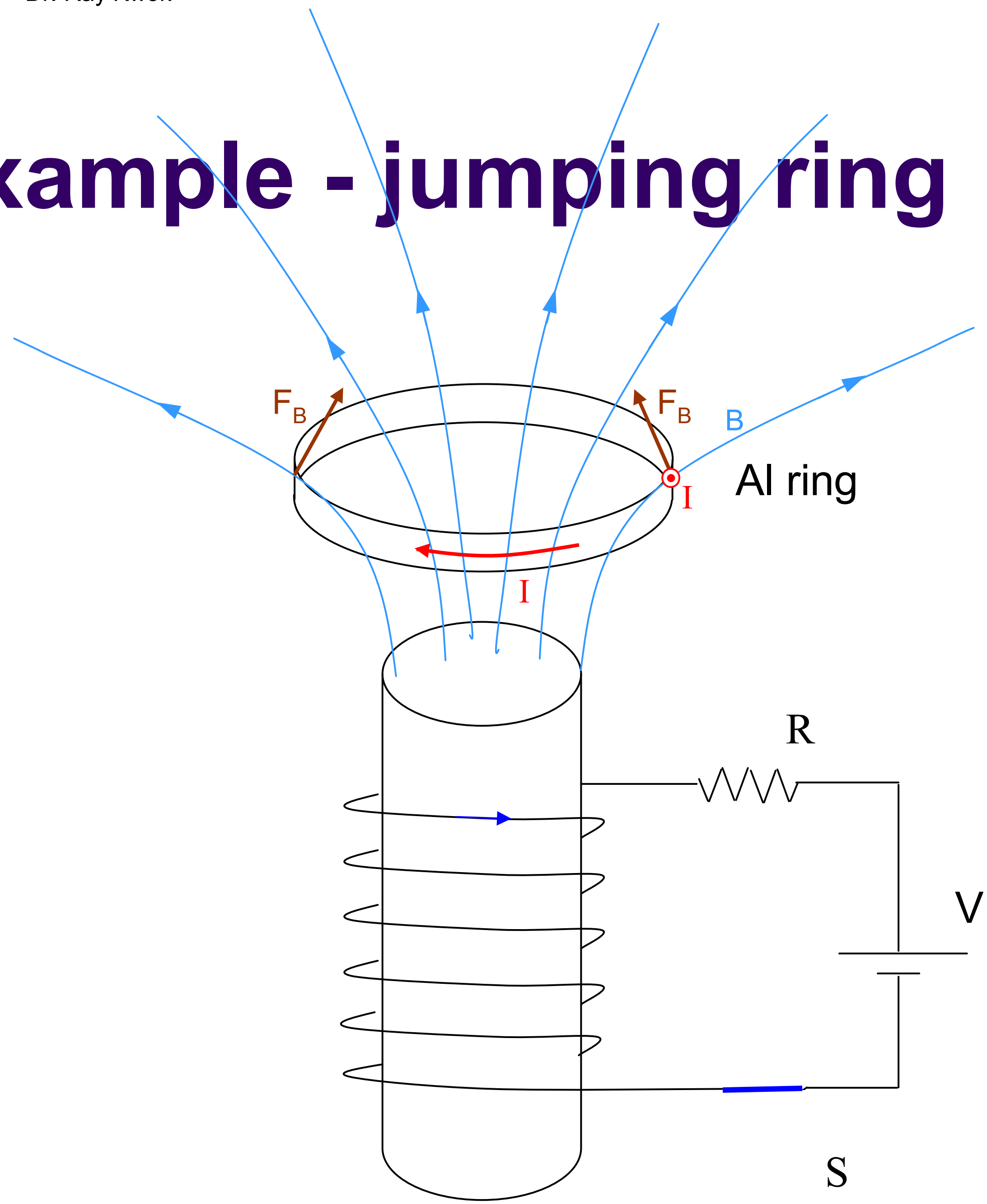


Heinrich Friedrich Emil Lenz
(1804-1865) Italy

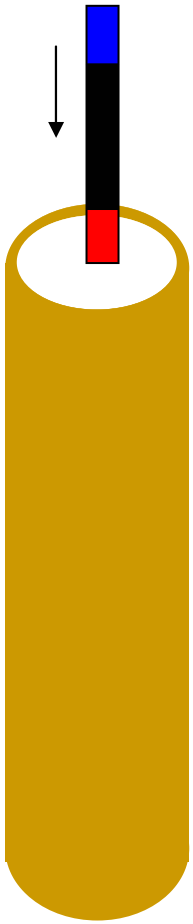
Example – moving magnet

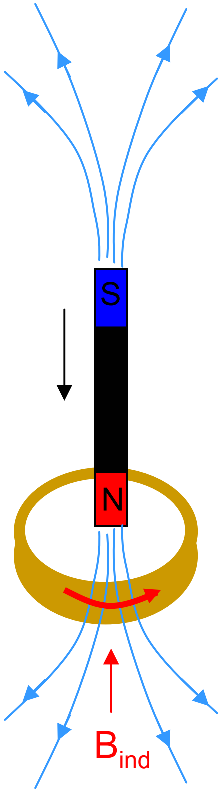


Example - jumping ring

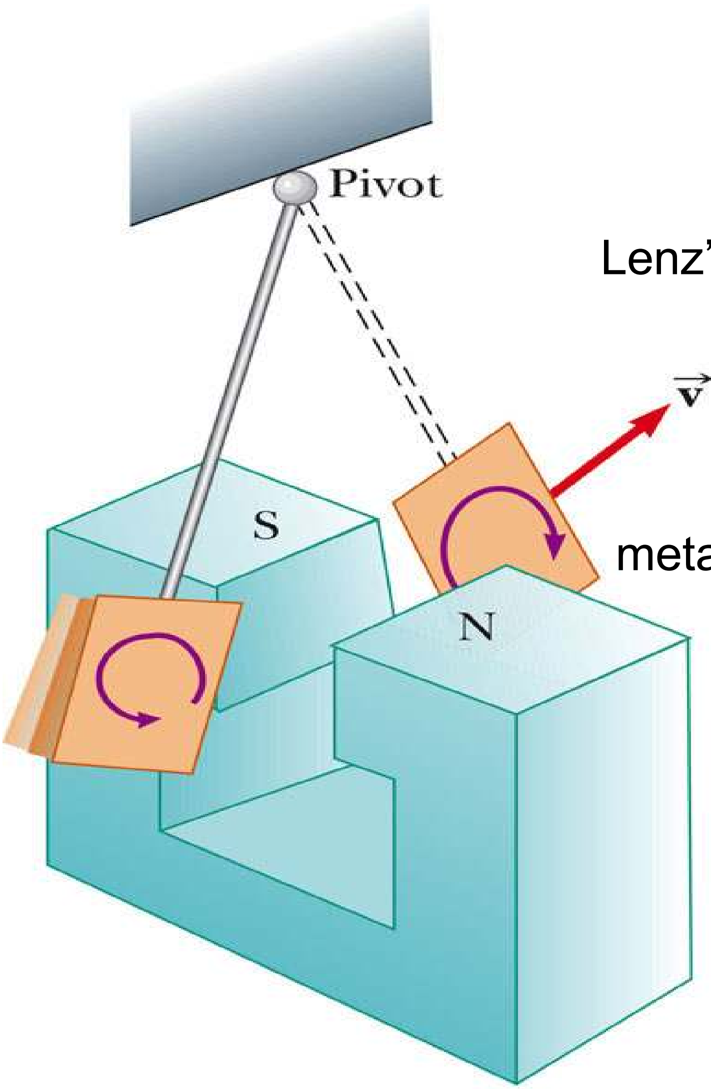


e.g. magnet in a copper tube



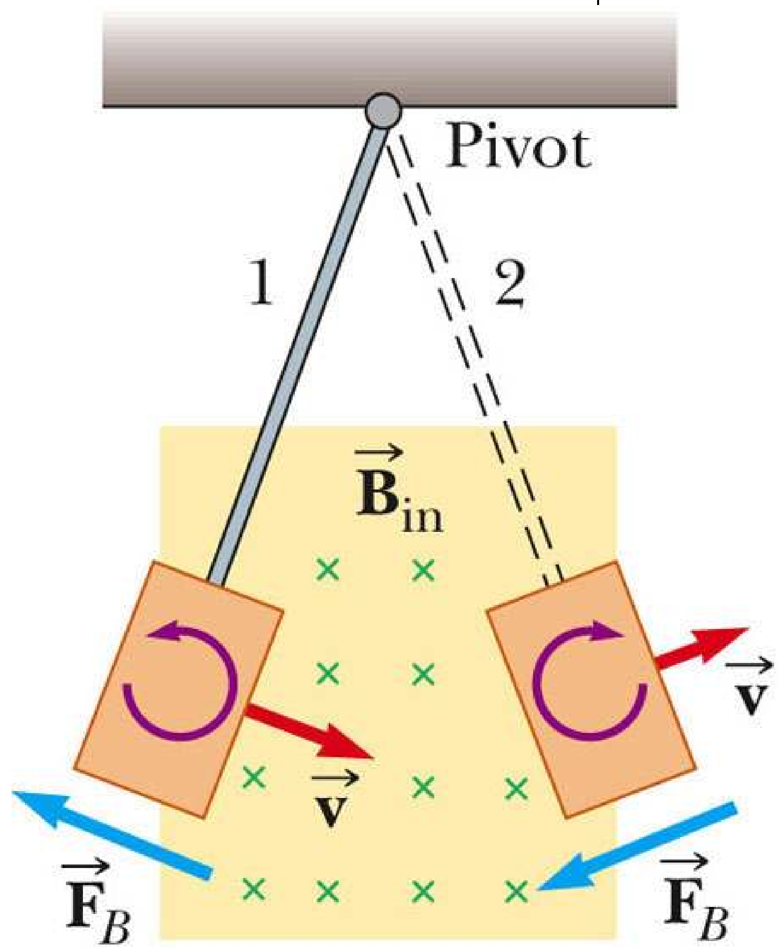


Eddy current - disc brake



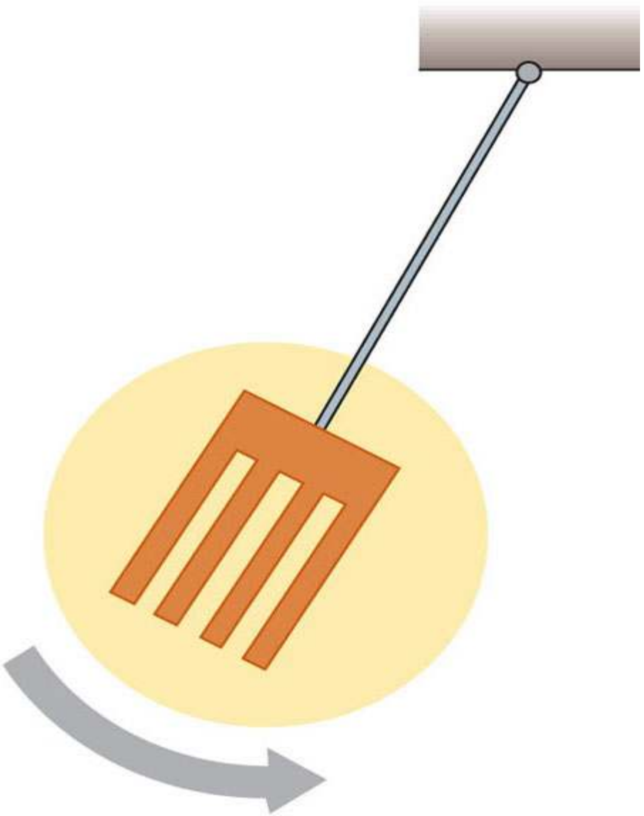
Lenz's Law

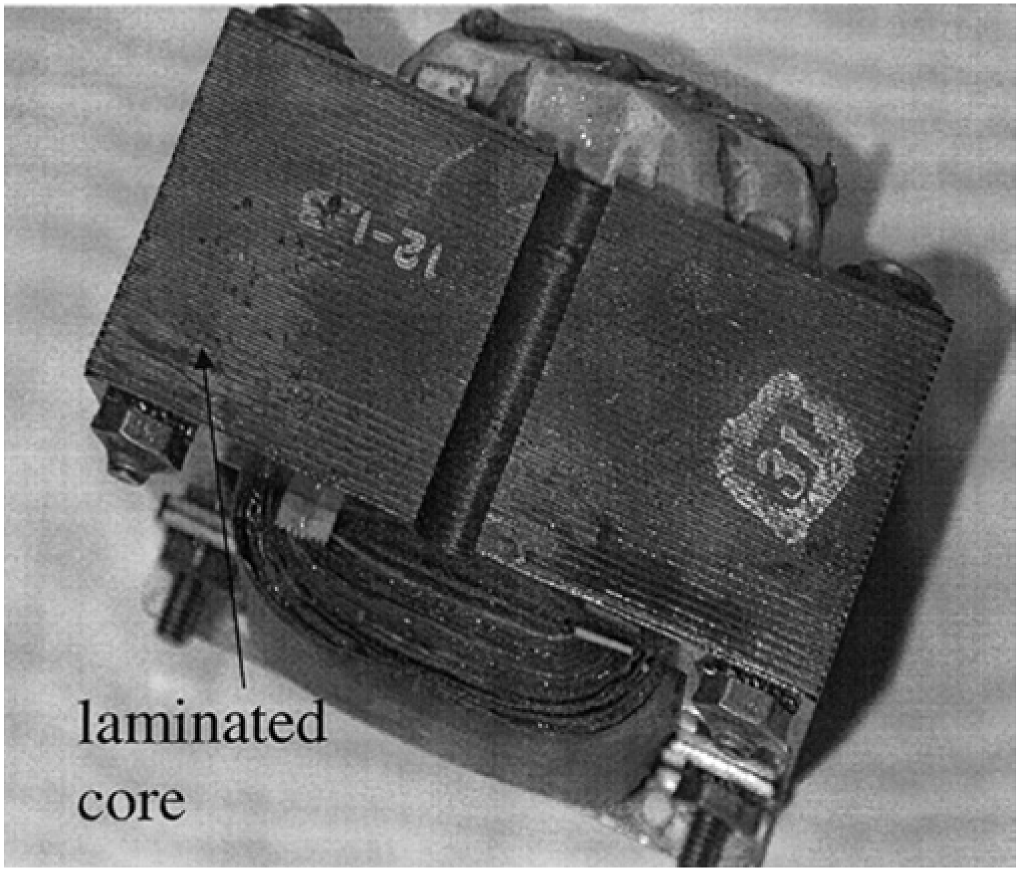
metal plate



$$\vec{F}_B = I\vec{L} \times \vec{B}$$

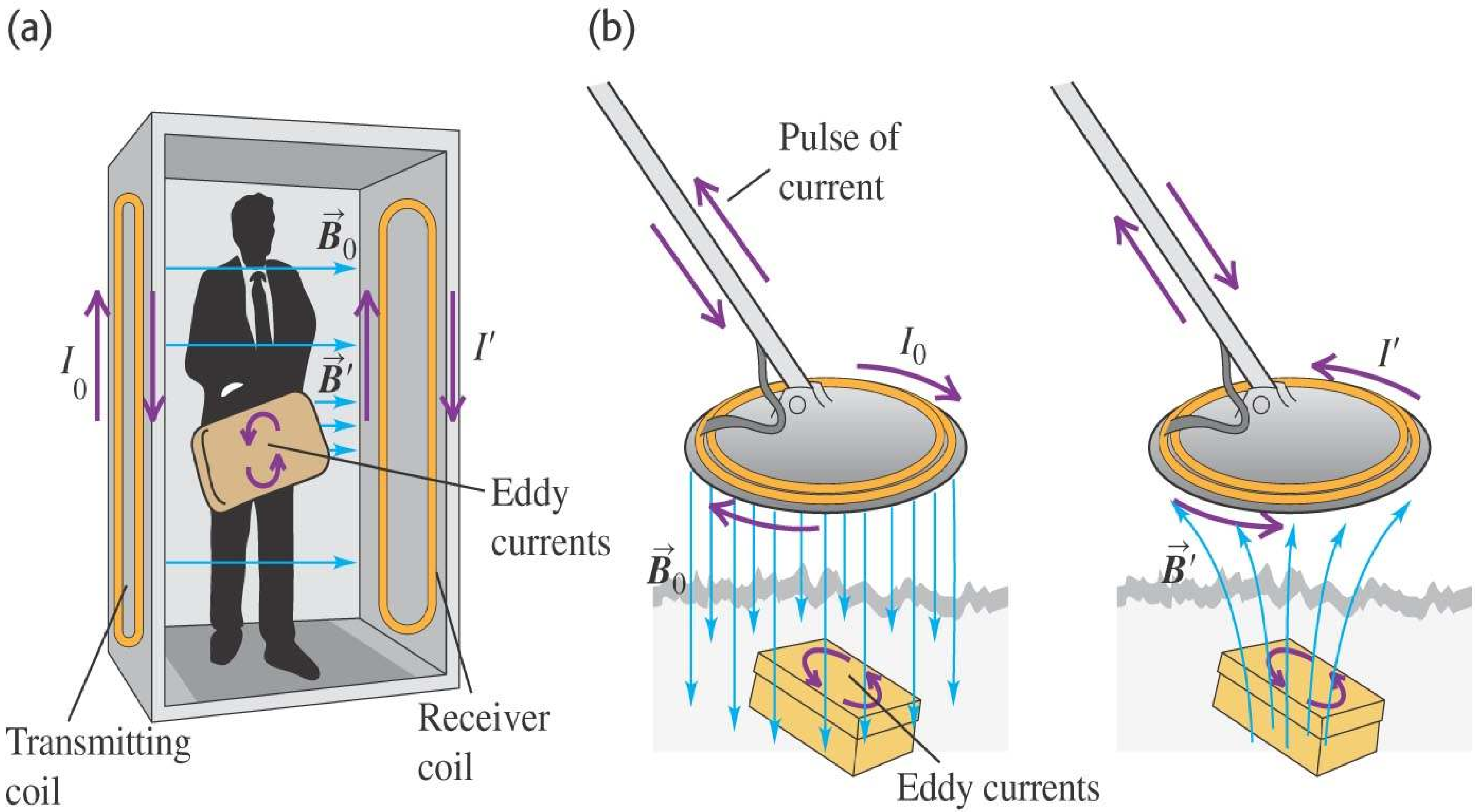
To reduce eddy current



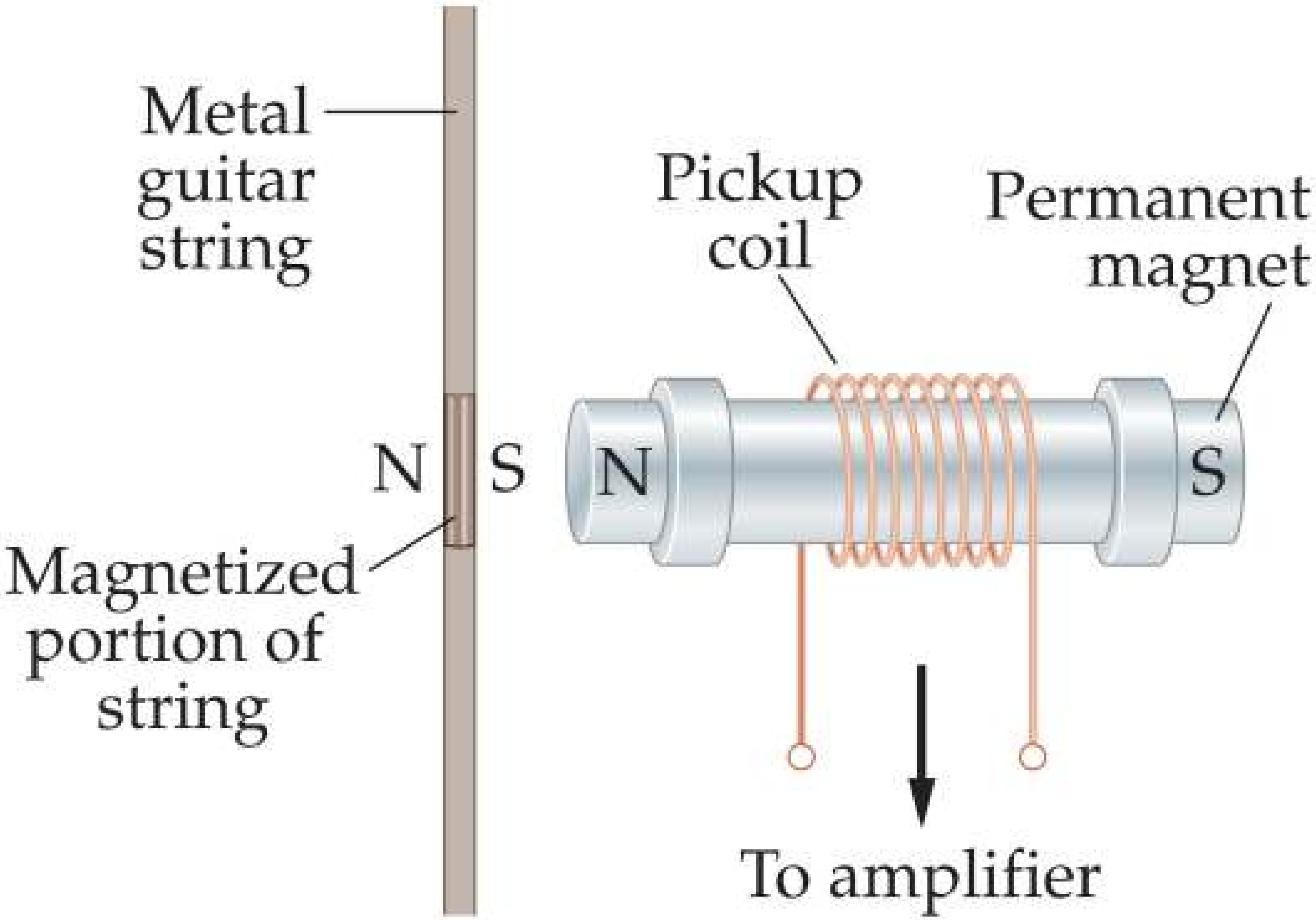


transformer

Example – metal detector

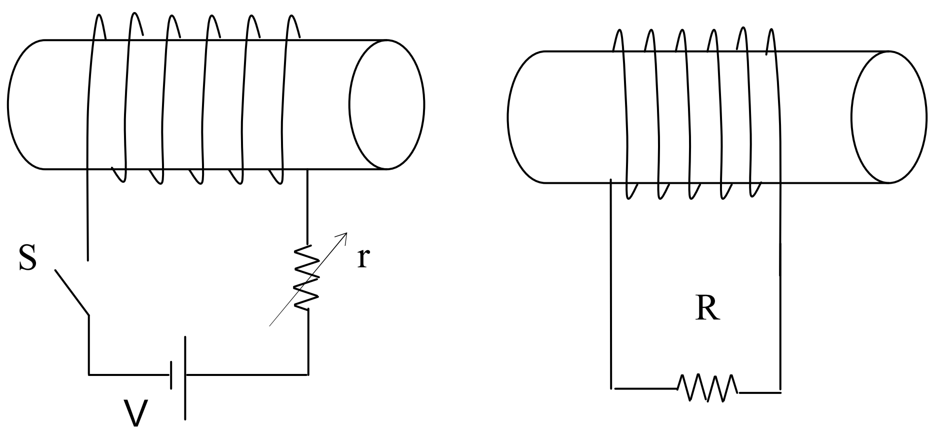


Example – guitar pickup



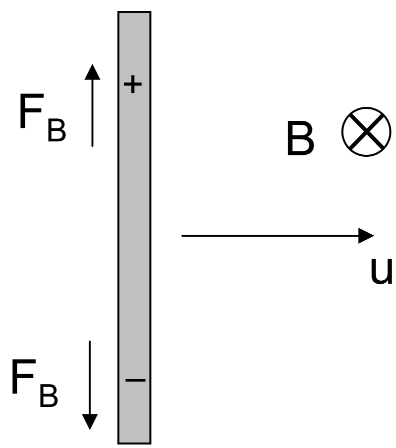
Question

Find the direction of the current in the resistor R shown in Figure at each the following steps: (a) at the instant the switch is closed, (b) after the switch has been closed for several minutes, (c) when the variable resistance r increases, (d) when the circuit containing R moving to the right, away from the other circuit, and (e) at the instant the switch is opened.



Right, 0, Left, Left, Left

Moving Conductor



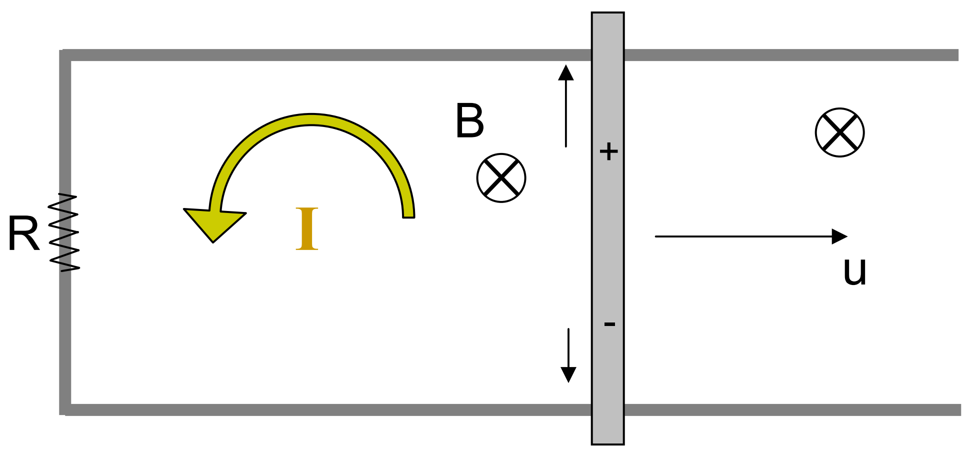
$$\vec{F}_E + \vec{F}_B = 0$$ separate the charges

saturate when $$q\vec{E} + q\vec{u}\times\vec{B} = 0$$ induced E

$$\vec{E} = -\vec{u}\times\vec{B}$$

induced $$\frac{V}{\ell} = -uB$$

emf $$V = -uB\ell$$ similar to Hall Effect



emf is "+" **u** x **B**

induced current
$I = |V/R| = B\ell u/R$

direction agrees w/ Lenz's Law

Generalized Faraday's Law

$$V_{emf} \equiv \oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi}{dt} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$ changing B(t), stationary loop

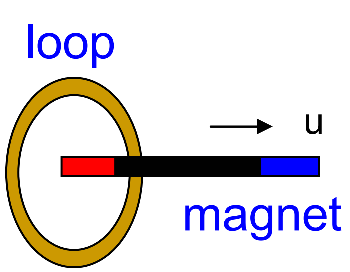
$$V_{emf} \equiv \oint \frac{\vec{F}_B}{q}\cdot d\vec{\ell} = \oint \vec{u}\times\vec{B}\cdot d\vec{\ell}$$ fixed B, moving loop

$$\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = \int\left(\frac{\partial \vec{B}}{\partial t} - \nabla\times(\vec{u}\times\vec{B})\right)\cdot d\vec{a} = \int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a} - \oint \vec{u}\times\vec{B}\cdot d\vec{\ell}$$ hw

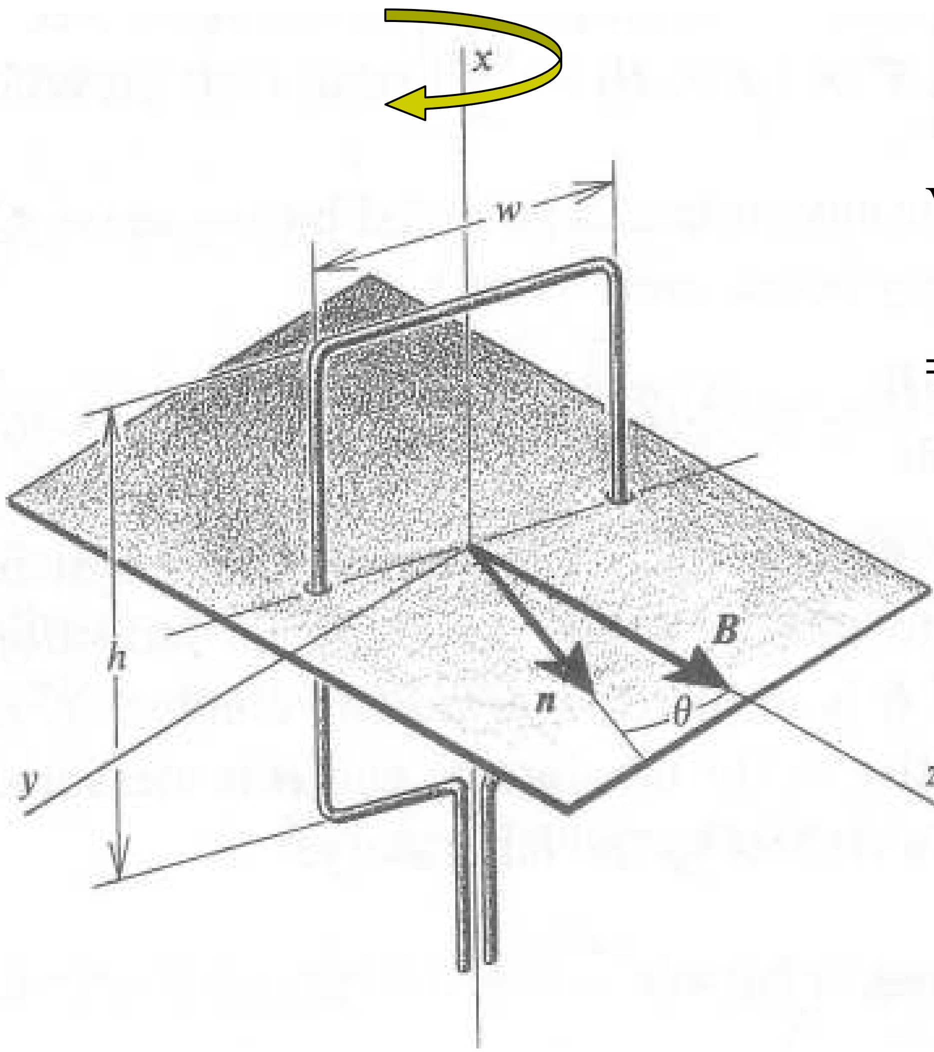
$$V_{emf} = -\frac{d\Phi}{dt} = \oint \vec{u}\times\vec{B}\cdot d\vec{\ell} - \int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

induced emf in a moving loop w.r.t. "stationary" B(t)

Einstein's Relativity:
move the loop, Lorentz force (magnetic) [motional emf]
move the magnet, induced emf – electric [transformer emf]



Example – rotating loop



$\theta = \omega t$ uniform B

$$V_{emf} = \oint \vec{u} \times \vec{B} \cdot d\vec{\ell} = 2(uB\sin\omega t)h$$

$$= 2\left(\frac{w}{2}\omega\right)Bh\sin\omega t = \omega BA\sin\omega t$$

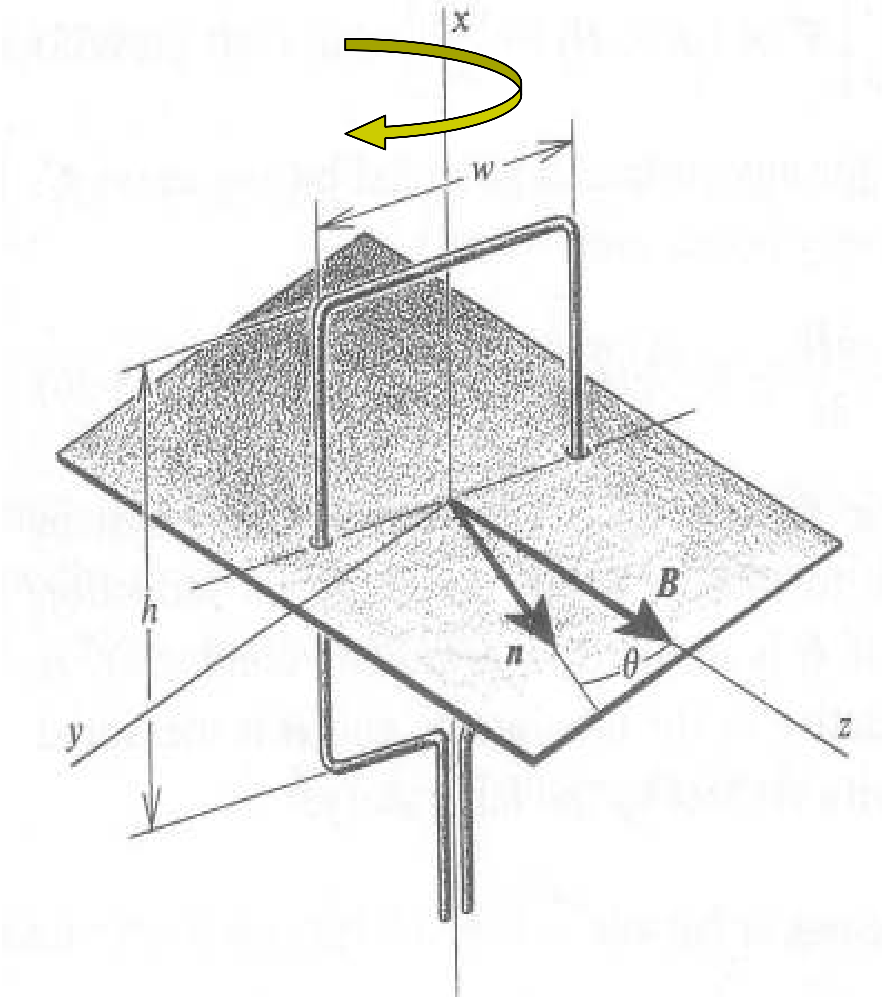
OR

$$V_{emf} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{a}$$

$$= -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$$

Direction of current (at this instance)??

Group Exercise

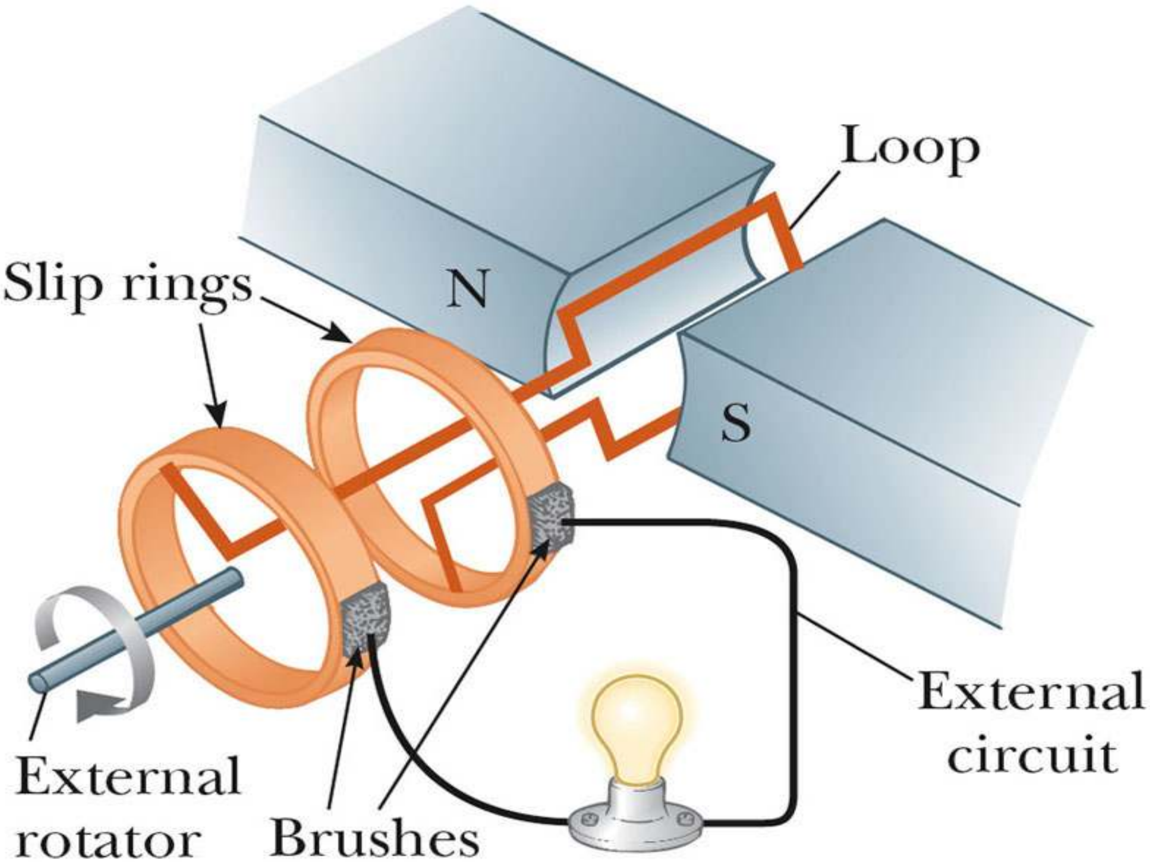


Find the induced emf in a rectangular loop rotating at an angular velocity ω in a magnetic field $B_o \sin\omega t$.

& the direction of induced current?

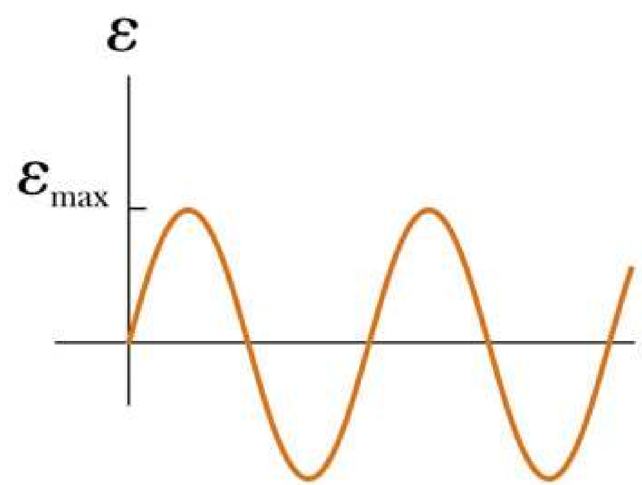
(Can you do this problem in 2 ways?)

AC generator

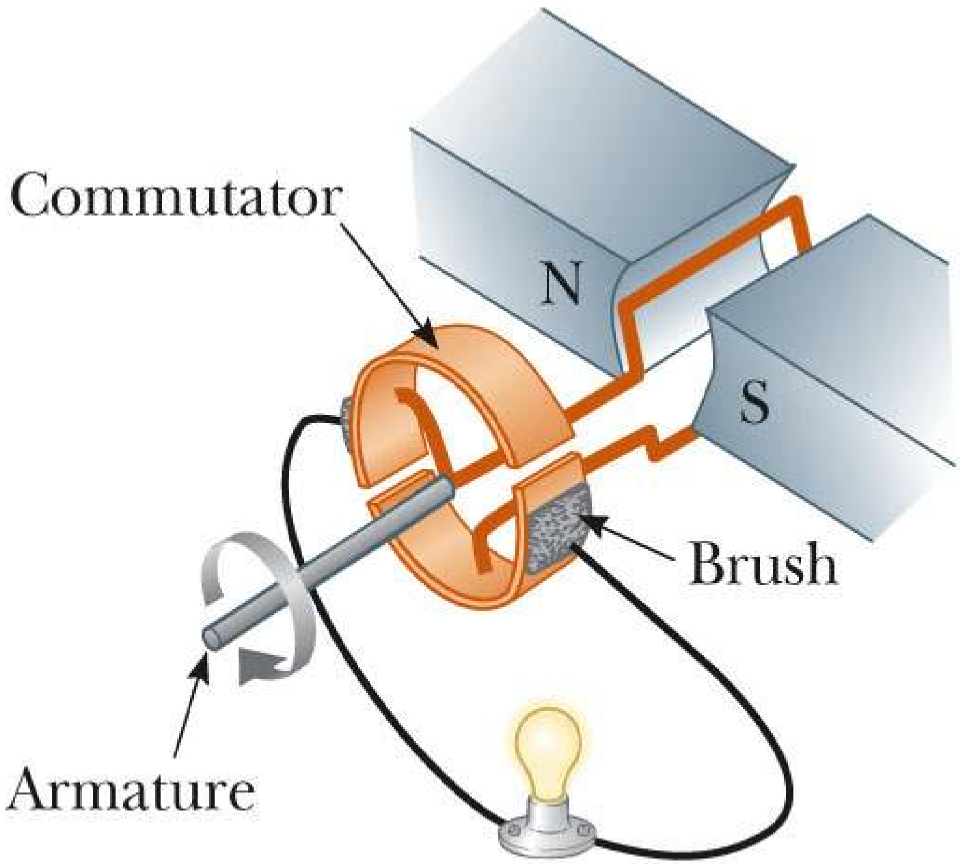


$$V_{emf} = -N\frac{d\Phi}{dt}$$

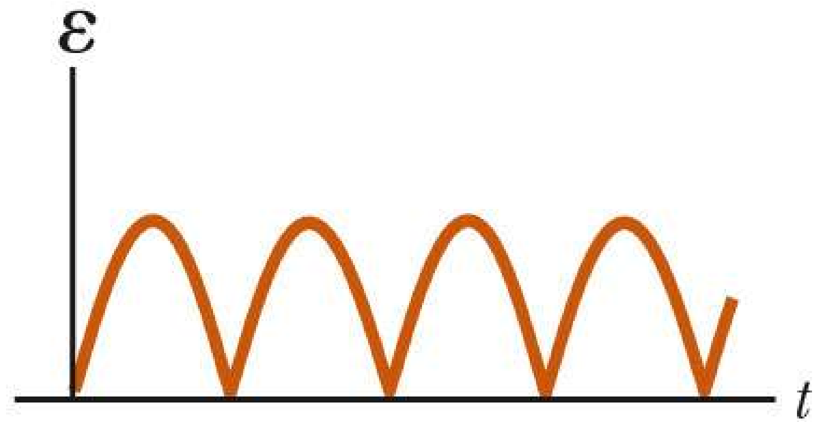
$$= N\omega BA\sin\omega t$$



DC generator

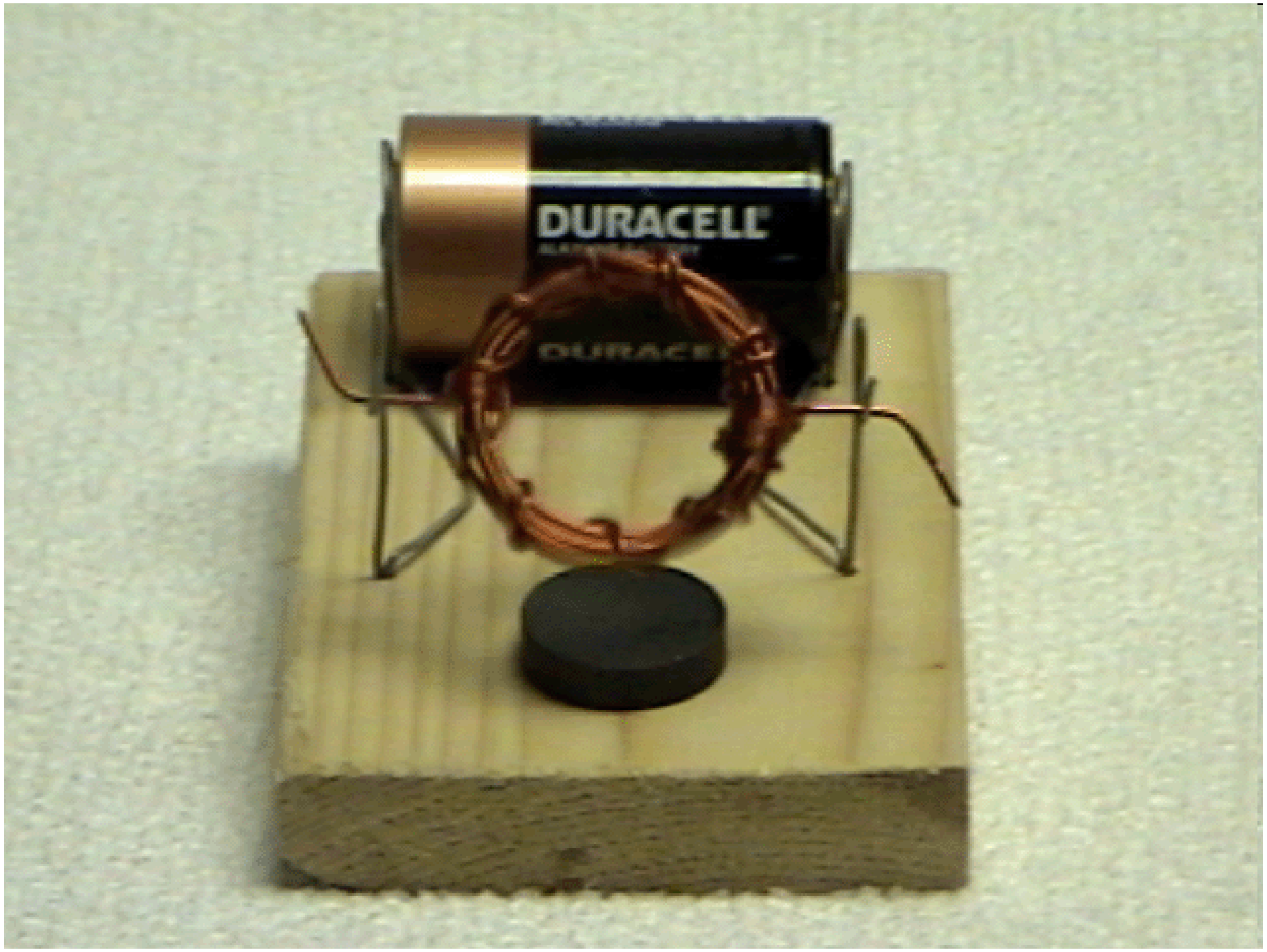


(a)



(b)

Electromotor



To turn faster, should we

1. use thicker wire?
2. use more turns?
3. make bigger loop?
4. use stronger magnet

Answer… (not counting friction)

$$V_{emf} = -N\frac{d\Phi}{dt} = N\omega BA\sin\omega t$$

$$|I| = \frac{N\omega BA}{R}$$

$$\vec{F}_B = I\vec{\ell}\times\vec{B}$$

$$\vec{\tau} = \vec{r}\times\vec{F}_B = \vec{m}\times\vec{B}$$ torque

$$\vec{m} = NIA\hat{n}$$ magnetic moment

$$\vec{\tau} = I\vec{\alpha}$$ I = moment of inertia

$$R = \rho\frac{\ell}{A'}$$ resistivity

$$m = \rho V = \rho A'\ell$$ mass density

1. use thicker wire?
same.
e.g. half R, double mass, same α

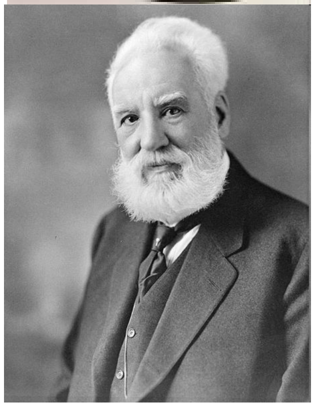
2. use more turns?
same.
double N, double R, same I,
double mm, but double inertia, same α.

3. use bigger loop? (same N)
same.
bigger A, more Φ
somewhat higher R, but still more $I \sim A/R \sim r$
mm~A^2/R~r^3, inertia~r^3, same α

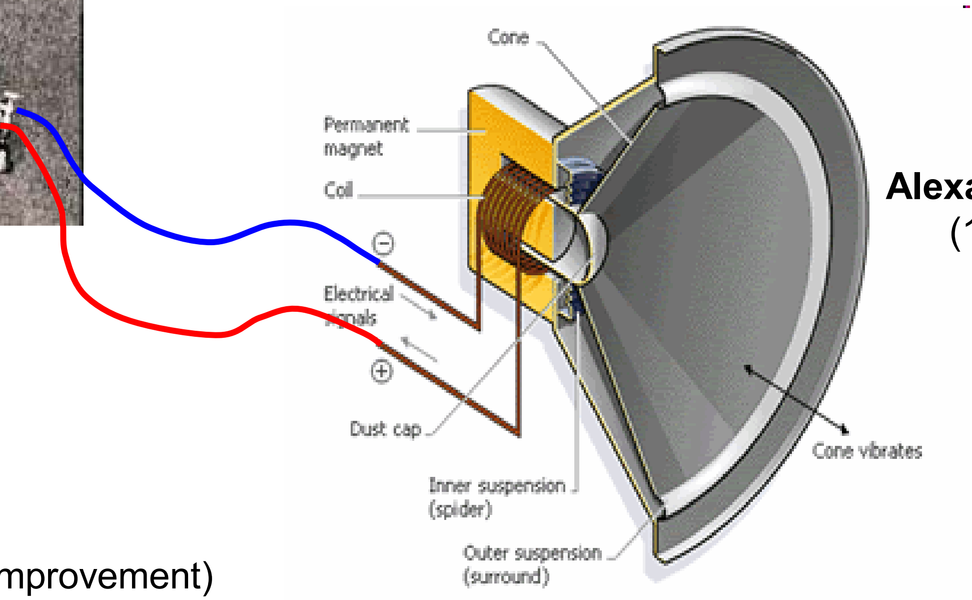
4. use stronger magnet?
YES
more I, m, τ, α

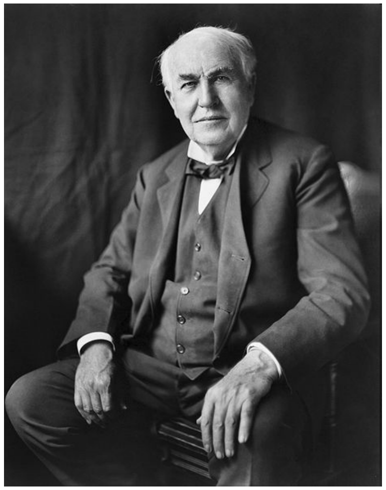
e.g. flash lights

Another important application – telephony / loudspeaker



(?? before him)
Alexander Graham Bell
(1847 –1922) UK

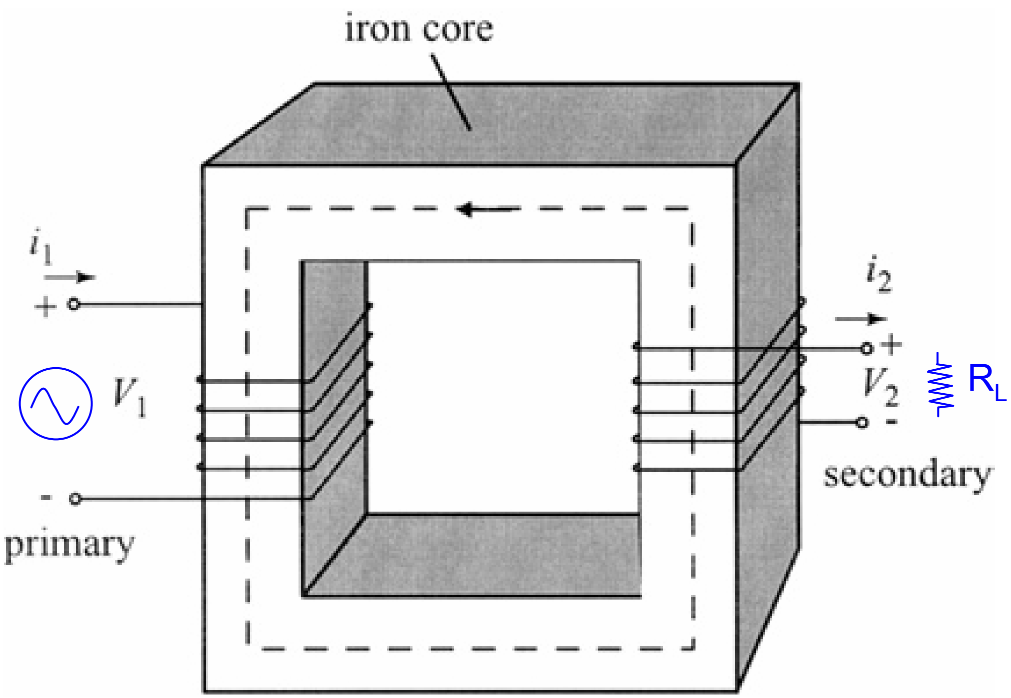




(practical improvement)
Thomas Alva Edison
(1847 –1931) USA

Ideal Transformer (AC)

Φ is confined in the core ($\mu = \infty$)



$$V_1 = -N_1 \frac{d\Phi}{dt}$$

$$V_2 = -N_2 \frac{d\Phi}{dt}$$

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

$$P_1 = P_2$$

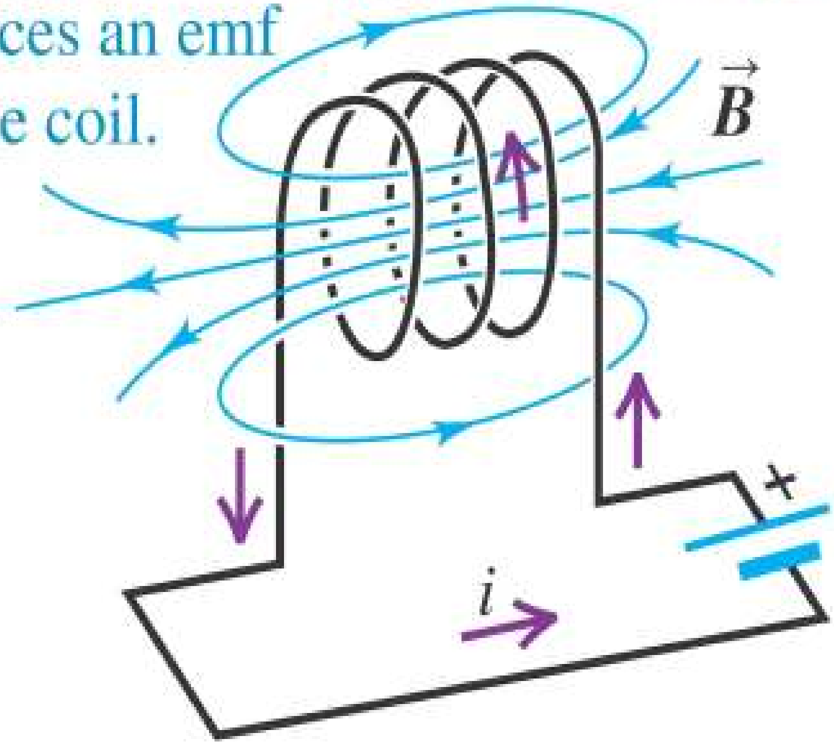
$$V_1 I_1 = V_2 I_2$$

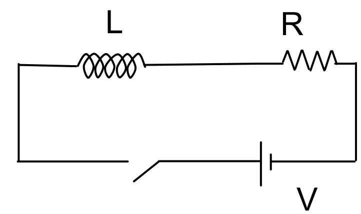
$$\frac{V_1}{V_2} = \boxed{\frac{I_2}{I_1} = \frac{N_1}{N_2}}$$

$$R_{in} = \frac{V_1}{I_1} = \frac{V_2}{I_2}\left(\frac{N_1}{N_2}\right)^2 \boxed{= \left(\frac{N_1}{N_2}\right)^2 R_L}$$

Self-Inductance

Self-inductance: If the current *i* in the coil is changing, the changing flux through the coil induces an emf in the coil.





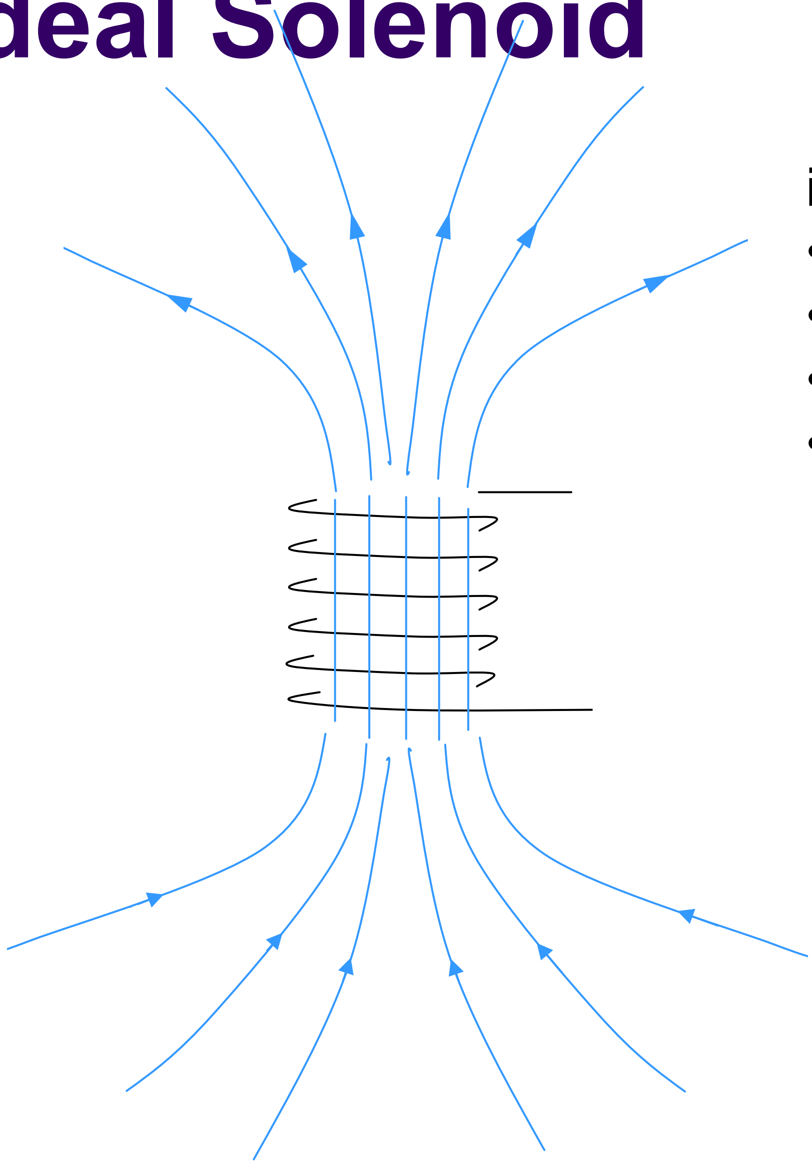
back emf causes I lags V

$V = IZ = I(R + j\omega L) = I\,|Z|e^{j\theta}$

$$L \equiv \frac{N\Phi}{I}$$

$$V = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

Ideal Solenoid



ideal:

- large N tightly wound
- no end effect
- uniform internal B
- zero external B in the vicinity

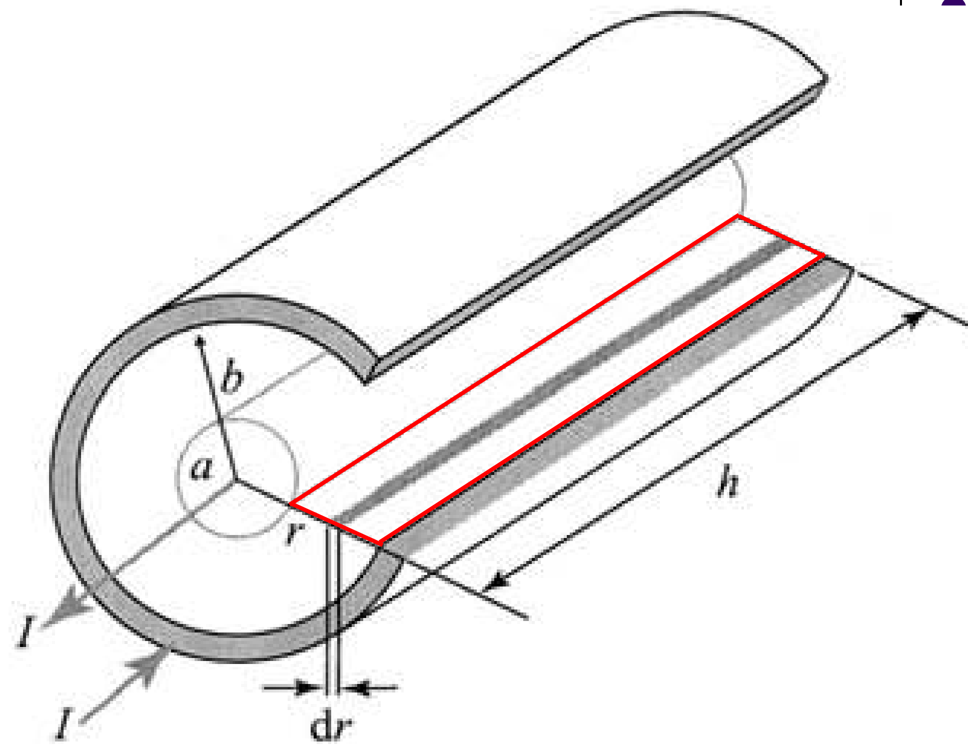
Ampere's Law:

$$B = \frac{\mu_o N I}{\ell} = \mu_o n I$$

$$\Phi = \int \vec{B} \cdot d\vec{a} = \mu_o n I A$$

$$L = \frac{N\Phi}{I} = \frac{N \mu_o n I A}{I} = \mu_o n^2 \ell A$$

Coaxial Cable



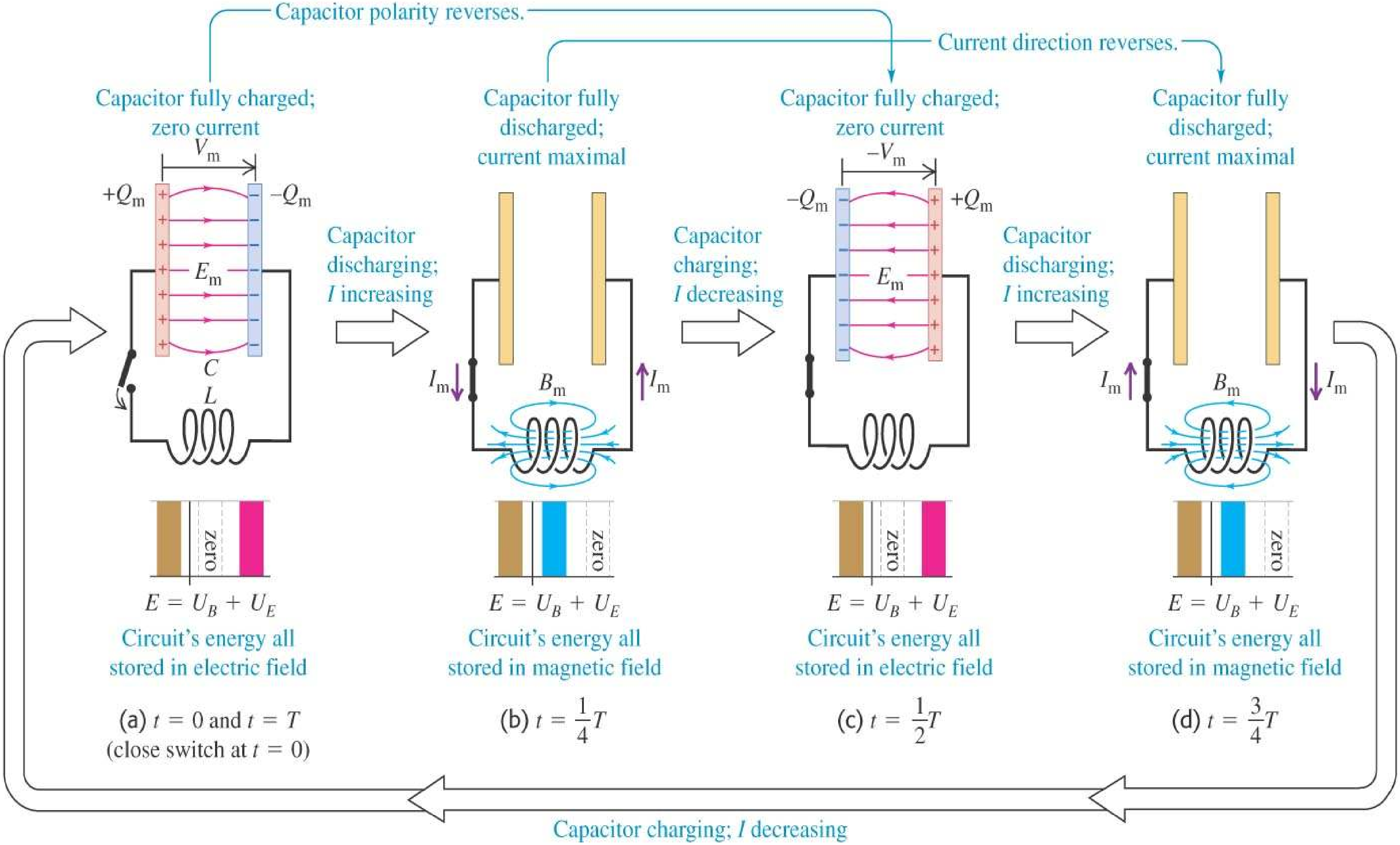
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I$$

$$\vec{B} = \frac{\mu_o I}{2\pi r}\hat{\phi}$$

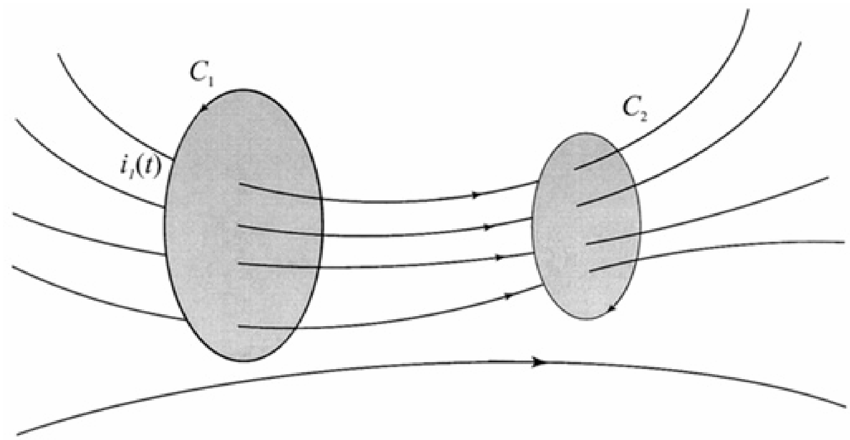
$$\Phi = \int \vec{B} \cdot d\vec{a} = \int_a^b \frac{\mu_o I}{2\pi r}\hat{\phi} \cdot \hat{\phi} h dr = \frac{\mu_o I h}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{\Phi}{I} = \frac{\mu_o h}{2\pi}\ln\left(\frac{b}{a}\right)$$

LC Resonator (Lenz's Law, later resonator…)



Mutual-Inductance



$$M_{12} \equiv \frac{\Phi_{12}}{I_1}$$

Φ_{12} is the flux through loop 2 due to the B generated by loop 1

$$\Phi_{12} = \int \vec{B}_1 \cdot d\vec{a}_2 = \int \nabla \times \vec{A}_1 \cdot d\vec{a}_2 = \oint \vec{A}_1 \cdot d\vec{\ell}_2$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} dV' = \frac{\mu_o}{4\pi} \oint \frac{I(r')}{|\vec{r} - \vec{r}\,'|} d\vec{\ell}\,'$$

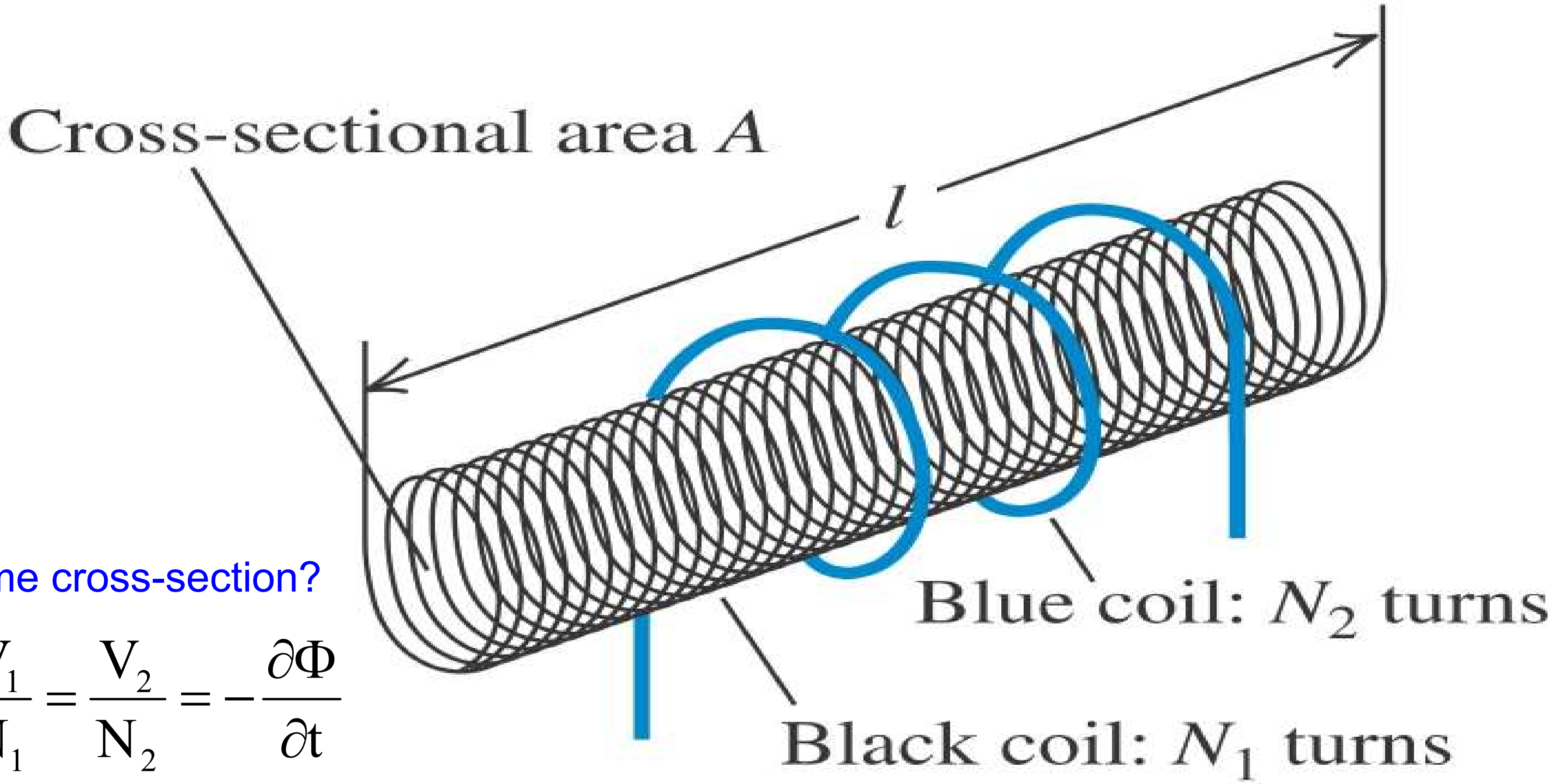
more than 1 turn?

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

$$\Phi_{12} = \frac{\mu_o}{4\pi} \oint\oint \frac{I_1}{r_{12}} d\vec{\ell}_1 \cdot d\vec{\ell}_2$$

$$M_{12} = \frac{\Phi_{12}}{I_1} = \frac{\mu_o}{4\pi} \oint\oint \frac{d\vec{\ell}_1 \cdot d\vec{\ell}_2}{r_{12}} = M_{21}$$

Transformer - Primary / Secondary coils



same cross-section?

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = -\frac{\partial \Phi}{\partial t}$$

$$\frac{N_2}{I_1} = \frac{N_1}{I_2} = \frac{M_{12}}{\Phi_{12}}$$

same equations

different cross-section? h.w.

Magnetic Energy

For an inductor …

$$W_m = \frac{1}{2}\int \frac{B^2}{\mu_o} dV$$

$$W_m = \frac{1}{2} L I^2$$

$$W_m = \frac{1}{2} I \Phi$$

For coupling circuits 1 & 2…

$$W_m = \frac{1}{2} I_1 \Phi_1 + \frac{1}{2} I_2 \Phi_2$$

$$\Phi_1 = \Phi_{11} + \Phi_{21}$$

$$\Phi_2 = \Phi_{22} + \Phi_{12}$$

$$W_m = \frac{1}{2} I_1 \Phi_{11} + \frac{1}{2} I_1 \Phi_{21} + \frac{1}{2} I_2 \Phi_{22} + \frac{1}{2} I_2 \Phi_{12}$$

$$\Phi_{11} = L_1 I_1$$

$$\Phi_{22} = L_2 I_2$$

$$\Phi_{12} = M_{12} I_1 = M I_1$$

$$\Phi_{21} = M_{21} I_2 = M I_2$$

$$W_m = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

Faraday's Law - differential

$$V_{emf} \equiv \oint \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$$\int \nabla\times\vec{E}\cdot d\vec{a} = -\int\left(\frac{\partial\vec{B}}{\partial t} - \nabla\times(\vec{u}\times\vec{B})\right)\cdot d\vec{a}$$

Stokes' theorem

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} + \nabla\times(\vec{u}\times\vec{B})$$

E is induced in moving medium
B is measured in stationary frame
curl operates in moving frame

$$\nabla\times\vec{E}(\vec{r}\,') = -\frac{\partial\vec{B}(\vec{r})}{\partial t} + \nabla'\times(\vec{u}\times\vec{B}(\vec{r}))$$

relativity, low f

$$\boxed{\nabla\times\vec{E}(\vec{r}) = -\frac{\partial\vec{B}(\vec{r})}{\partial t}}$$

for stationary loop (rest frame)
or localized, RF EM wave (microscopic)

Electrodynamics

$$\left\{\begin{aligned}
&\oint \vec{E}\cdot d\vec{a} = \frac{Q_t}{\varepsilon_o} && \text{Gauss's Law}\\
&\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi}{dt} && \text{Faraday's Law}\\
&\oint \vec{B}\cdot d\vec{a} = 0 && \text{No magnetic charge}\\
&\oint \vec{B}\cdot d\vec{\ell} = \mu_o I_t && \text{Ampere's Law}
\end{aligned}\right\}$$

$$\left\{\begin{aligned}
\nabla\cdot\vec{D} &= \rho_f\\
\nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\\
\nabla\cdot\vec{B} &= 0\\
\nabla\times\vec{H} &= \vec{J}_f
\end{aligned}\right\}$$

Electric Potential

static $\nabla \times \vec{E} = 0$

define $\vec{E} \equiv -\nabla V$

such that $\nabla \times \nabla V = 0$

$\nabla \cdot \vec{B} = 0$

define $\vec{B} \equiv \nabla \times \vec{A}$

such that $\nabla \cdot \nabla \times \vec{A} = 0$

dynamic $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} \nabla \times \vec{A}$

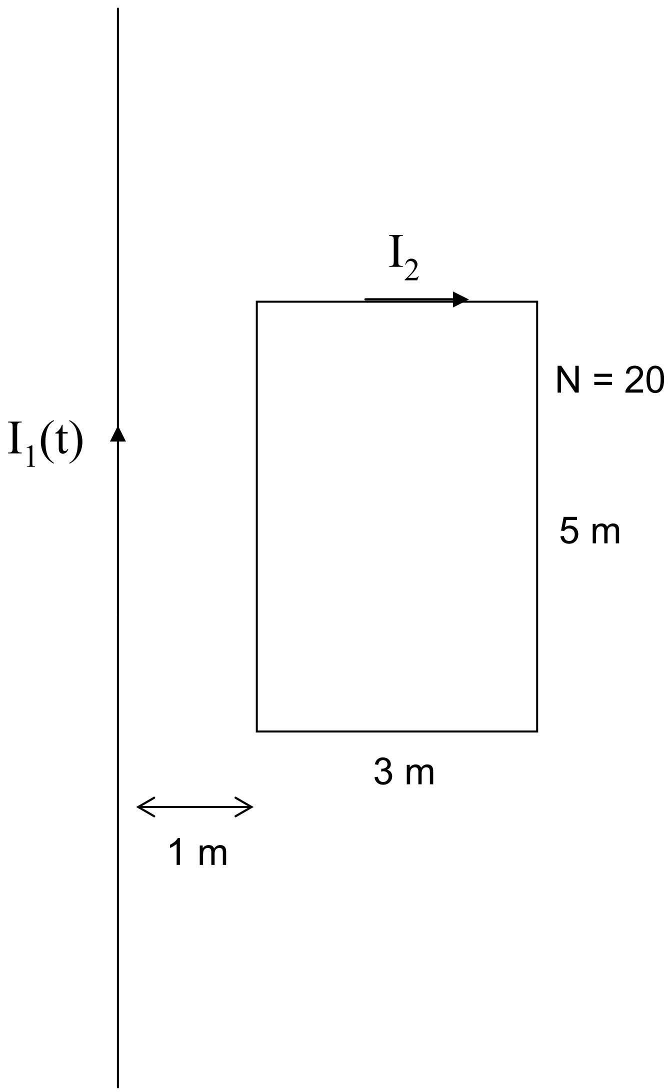
$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} \equiv -\nabla V$$

$$\boxed{\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}}$$

Need both V and A to find E !!!!!

Group Exercise



Rectangular loop of 20 turns is placed at 1 m away from a long current-carrying wire as shown.

$I_1(t) = 2\cos(60t)$ (A)
Resistivity of the wire in the loop is 4 Ω/m.

Find I_2.