Plane Electromagnetic Waves

EE142

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•reference:
Fundamentals of Engineering Electromagnetics, David K. Cheng (Addison-Wesley)
Electromagnetics for Engineers, Fawwaz T. Ulaby (Prentice Hall)

Source-free Maxwell Equations

$$\varepsilon\nabla\cdot\vec{E}=\rho_f$$
$$\nabla\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t}$$
$$\nabla\cdot\vec{H}=0$$
$$\nabla\times\vec{H}=\vec{J}_f+\varepsilon\frac{\partial\vec{E}}{\partial t}$$

in source-free medium
(homogeneous,
linear, isotropic)

$\rho = 0,\ J = 0$

→

$$\nabla\cdot\vec{E}=0$$
$$\nabla\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t}$$
$$\nabla\cdot\vec{H}=0$$
$$\nabla\times\vec{H}=\varepsilon\frac{\partial\vec{E}}{\partial t}$$

Wave Equation

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla \times \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) = -\mu \frac{\partial}{\partial t}\left(\nabla \times \vec{H}\right) = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

$$\boxed{\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$$

plane wave equation with $v^2 = 1/\mu\varepsilon$

Traveling Wave

$$f(x \pm vt) \equiv f(u)$$

reverse / forward traveling wave

$$\frac{\partial f}{\partial x} = f'(u)\frac{\partial u}{\partial x} = f'(u)$$

$$\frac{\partial^2 f}{\partial x^2} = f''(u)\frac{\partial u}{\partial x} = f''(u)$$

$$\frac{\partial f}{\partial t} = f'(u)\frac{\partial u}{\partial t} = \pm v f'(u)$$

$$\frac{\partial^2 f}{\partial t^2} = \pm v f''(u)\frac{\partial u}{\partial t} = v^2 f''(u)$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

wave equation

note:

$$x \pm vt = \frac{1}{k}\left(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{\lambda}vt\right)$$

$$= \frac{1}{k}(kx \pm 2\pi ft)$$

$$= \pm\frac{1}{k}(\omega t \pm kx)$$

$$f(x \pm vt) = f(\omega t \pm kx)$$

$$f(\omega t - \vec{k} \cdot \vec{r})$$ (3D)

Plane Wave

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{E}(\vec{r}, t) = \vec{E}_o e^{j(\omega t - \vec{k}\cdot\vec{r})}$$

$$\nabla^2 \vec{E} = -k^2 \vec{E}$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \vec{E}$$

$$-k^2 = -\omega^2 \mu\varepsilon$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$\frac{\omega}{k} = v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Similarly for B or H field

$$\nabla^2 \vec{H} = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\vec{H}(\vec{r}, t) = \vec{H}_o e^{j(\omega t - \vec{k}\cdot\vec{r})}$$

Source-free EM wave

$$\vec{E}(\vec{r},t) = \hat{x}E_o e^{j(\omega t - kz)}$$ V/m in vacuum with no current source.

What is **H** ?

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_o e^{j(\omega t - kz)} & 0 & 0 \end{vmatrix}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\nabla \times \vec{E} = \hat{y}\left\{-jkE_o e^{j(\omega t - kz)}\right\}$$

$$\vec{H}(\vec{r},t) = \hat{y}\frac{kE_o}{\omega\mu} e^{j(\omega t - kz)}$$

$$\nabla \times \vec{E} = \hat{y}\left\{-jkE_o e^{j(\omega t - kz)}\right\}$$

$$\vec{H}(\vec{r},t) = \hat{y}\frac{kE_o}{\omega\mu} e^{j(\omega t - kz)}$$

$$\frac{k}{\omega\mu} = \frac{1}{f\lambda\mu} = \frac{1}{v\mu} = \frac{\sqrt{\mu\varepsilon}}{\mu} = \sqrt{\frac{\varepsilon}{\mu}} \equiv \frac{1}{\eta}$$

$$\vec{H}(\vec{r},t) = \hat{y}\frac{E_o}{\eta} e^{j(\omega t - kz)}$$

η = wave Impedance

$$\vec{B} = \mu\vec{H} = \hat{y}\frac{\mu E_o}{v\mu} e^{j(\omega t - kz)}$$

$$\vec{B}(\vec{r},t) = \hat{y}\frac{E_o}{v} e^{j(\omega t - kz)}$$

EM wave properties

$$\vec{E}(\vec{r},t) = \hat{x}E_o e^{j(\omega t - kz)}$$

$$\vec{H}(\vec{r},t) = \hat{y}\frac{E_o}{\eta} e^{j(\omega t - kz)}$$

$$\vec{B}(\vec{r},t) = \hat{y}\frac{E_o}{v} e^{j(\omega t - kz)}$$

(1) E & H are in phase.

(2) $\vec{E} \perp \vec{H} \perp \vec{k}$

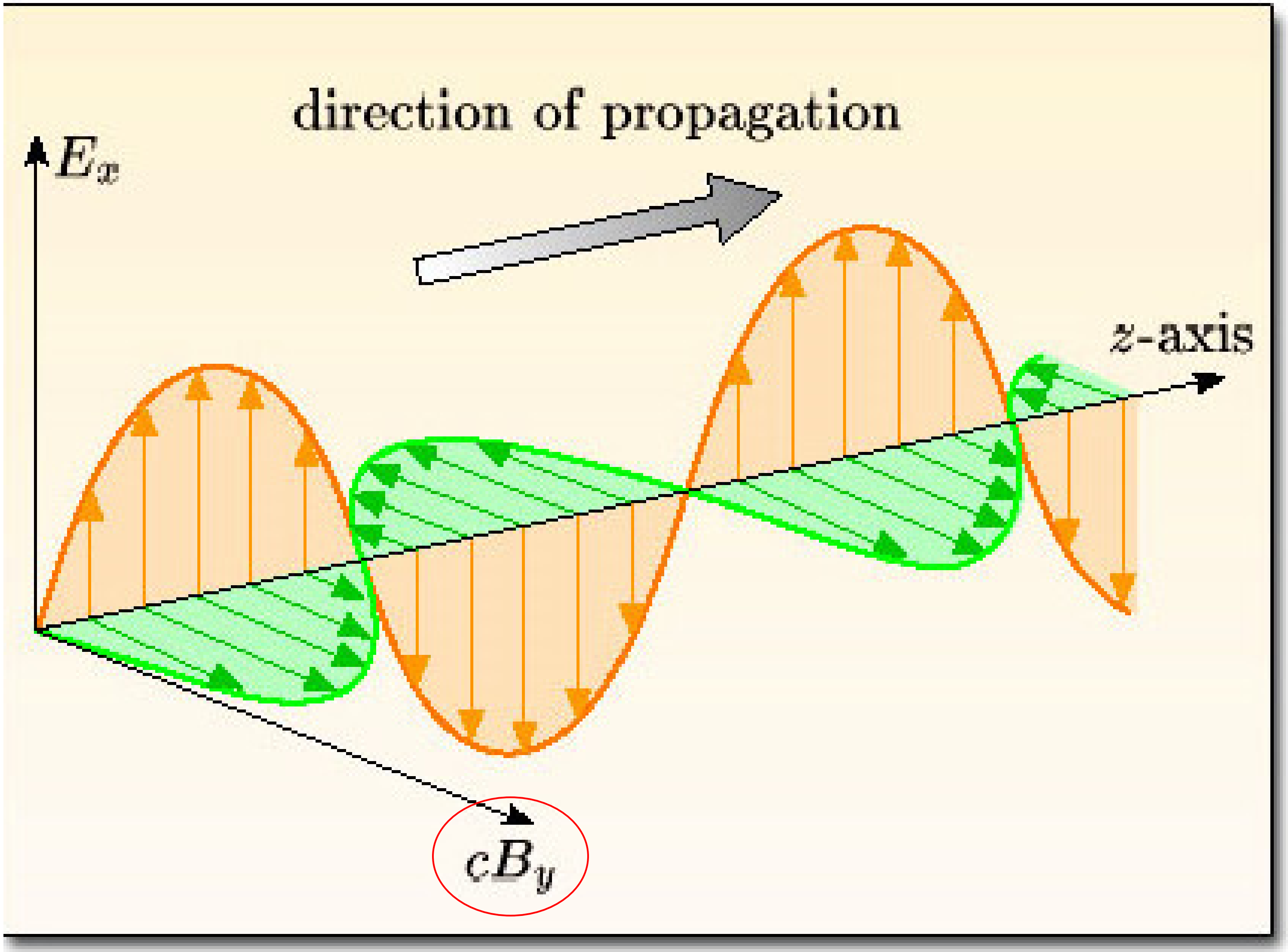
$$\hat{k} = \hat{E} \times \hat{H}$$

Index of refraction

(3) $\dfrac{E_o}{B_o} = v = \dfrac{c}{n} = \dfrac{c}{\sqrt{\varepsilon_r \mu_r}}$

$$\frac{E_o}{H_o} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Electromagnetic wave



Earlier exercise

$\vec{H}(\vec{r},t) = \hat{x}0.01\cos(900t + \beta z)$ A/m in vacuum with no current source.

What is f, λ, direction of propagation, **E** ?

$\omega = 2\pi f = 900$, $f = 143$ Hz

$\lambda = c/f = (3 \times 10^8)/(143) = 2094$ km

propagate to the –z direction

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

E(**r**) = $\mathbf{a}_y$ 3.77 cos[900t + 3(10^{-6})z] V/m

(1) E & H are in phase.

(2) $\vec{E} \perp \vec{H} \perp \vec{k}$

$$\hat{k} = \hat{E} \times \hat{H}$$

(3) $\dfrac{E_o}{B_o} = v$

$$\frac{E_o}{H_o} = \eta$$

$\eta_o = 377\ \Omega$

Example

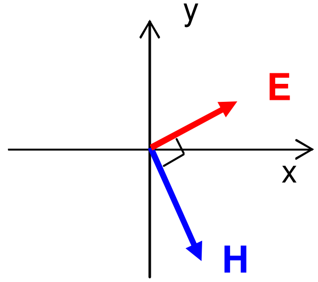
$\vec{H}(\vec{r},t) = (\hat{x}0.01 - \hat{y}0.02)\sin(10^9 t + \beta z - 0.1)$ A/m. What is **E** ?

$\beta = \omega/c = 10^9/(3 \times 10^8) = 3.33$ rad/m

(1) E & H are in phase. $\vec{E}(\vec{r},t) = \vec{E}_o \sin(10^9 t + 3.33z - 0.1)$ V/m

(2) $\vec{E} \perp \vec{H} \perp \vec{k}$

$\hat{k} = \hat{E} \times \hat{H}$



$$\hat{E} = \frac{2\hat{x} + \hat{y}}{\sqrt{5}} = 0.894\hat{x} + 0.447\hat{y}$$

(3) $\frac{E_o}{B_o} = v$

$\frac{E_o}{H_o} = \eta$

$\eta_o = 377\ \Omega$

$H_o^2 = 0.01^2 + 0.02^2 \rightarrow H_o = 0.022 = E_o/\eta_o = E_o/377$
$E_o = 8.3$ V/m

$$\vec{E}(\vec{r},t) = 8.3(\hat{x}0.894 + \hat{y}0.447)\sin(10^9 t + 3.33z - 0.1)$$

$$\vec{E}(\vec{r},t) = (\hat{x}7.4 + \hat{y}3.7)\sin(10^9 t + 3.33z - 0.1) \quad \text{V/m}$$

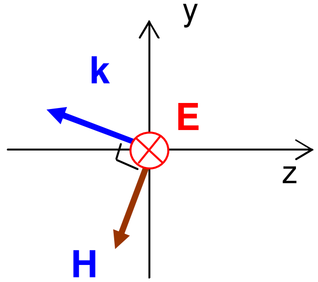
Exercise

$\vec{E}(\vec{r},t) = \hat{x}20\cos(10^8 t + 3z - y)$ V/m in non-magnetic material.
What is **H** ?

(1) E & H are in phase. $\vec{H}(\vec{r},t) = \vec{H}_o \cos(10^8 t + 3z - y)$ A/m

(2) $\vec{E} \perp \vec{H} \perp \vec{k}$

$\hat{k} = \hat{E} \times \hat{H}$



$\vec{k} = -3\hat{z} + \hat{y}$

$\hat{H} = \frac{-3\hat{y} - \hat{z}}{\sqrt{10}} = -0.949\hat{y} - 0.316\hat{z}$

(3) $\frac{E_o}{B_o} = v$

$\frac{E_o}{H_o} = \eta$

$\eta_o = 377\ \Omega$

$v = \omega/k = 10^8/\sqrt{10} = 3.16 \times 10^7$ m/s

$v = c/n = c/\sqrt{\varepsilon_r} \rightarrow \varepsilon_r = (c/v)^2 = 90$

$\eta = \sqrt{(\mu/\varepsilon)} = \eta_o/\sqrt{\varepsilon_r} = 377/\sqrt{90} = 39.7$

$H_o = E_o/\eta = 20/39.7 = 0.5$

$\vec{H}(\vec{r},t) = 0.5(-\hat{y}0.949 - \hat{z}0.316)\cos(10^8 t + 3z - y)$

$\vec{H}(\vec{r},t) = -(\hat{y}0.475 + \hat{z}0.158)\cos(10^8 t + 3z - y)$ A/m

EM wave properties - generalized

(1) E & H are in phase.

$$\vec{H} = \frac{1}{\eta}\hat{k}\times\vec{E}$$

(2) $\vec{E} \perp \vec{H} \perp \vec{k}$

$$\hat{k} = \hat{E}\times\hat{H}$$

Index of refraction

$$\vec{E} = \eta\vec{H}\times\hat{k}$$

(3) $$\frac{E_o}{B_o} = v = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

$$\frac{E_o}{H_o} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Earlier exercise (free space)

$$\vec{H}(\vec{r},t) = (\hat{x}0.01 - \hat{y}0.02)\sin(10^9 t + \beta z - 0.1)$$ A/m. What is **E** ?

$\beta = \omega/c = 10^9/(3 \times 10^8) = 3.33$ rad/m

$$\vec{E} = \eta \vec{H} \times \hat{k}$$

$$\vec{E} = 377(\hat{x}0.01 - \hat{y}0.02)\sin(\omega t + \beta z - 0.1) \times (-\hat{z})$$

$$\vec{E} = (\hat{y}3.77 + \hat{x}7.54)\sin(10^9 t + 3.33z - 0.1)$$ V/m.

Earlier exercise

$\vec{E}(\vec{r},t) = \hat{x}20\cos(10^8 t + 3z - y)$ V/m in non-magnetic material.

What is **H** ?

$$\vec{k} = -3\hat{z} + \hat{y}$$

v = ω/k = $10^8/\sqrt{10}$ = 3.16×10^7 m/s

v = c/n = $c/\sqrt{\varepsilon_r} \rightarrow \varepsilon_r = (c/v)^2 = 90$

$\eta = \sqrt{(\mu/\varepsilon)} = \eta_o/\sqrt{\varepsilon_r} = 377/\sqrt{90} = 39.7$

$$\vec{H} = \frac{1}{\eta}\hat{k}\times\vec{E}$$

$$\vec{H} = \frac{1}{39.7}\left(\frac{-3\hat{z}+\hat{y}}{\sqrt{10}}\right)\times\left[\hat{x}20\cos\left(10^8 t + 3z - y\right)\right]$$

$$\vec{H} = \left(-\hat{y}0.478 - \hat{z}0.159\right)\cos\left(10^8 t + 3z - y\right) \quad \text{A/m}$$

Poynting Theorem

the work-energy theorem for electrodynamics

Work done on a charge δq by EM force:

$$dW = \vec{F}\cdot d\vec{\ell} = \delta q\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot \vec{v}dt$$

$$dW = \delta q\vec{E}\cdot\vec{v}dt$$

$$dW = (\rho\delta V)\vec{E}\cdot\vec{v}dt = \delta V\vec{E}\cdot\vec{J}dt$$

$$\frac{dW}{dt} = \int \vec{E}\cdot\vec{J}dV$$

$$\nabla\times\vec{H} = \vec{J}_f + \varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\vec{E}\cdot\vec{J} = \vec{E}\cdot\nabla\times\vec{H} - \varepsilon\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\left(\vec{E}\times\vec{H}\right) = \vec{H}\cdot\nabla\times\vec{E} - \vec{E}\cdot\nabla\times\vec{H}$$ **(math)**

$$\vec{E}\cdot\nabla\times\vec{H} = \vec{H}\cdot\left(-\mu\frac{\partial\vec{H}}{\partial t}\right) - \nabla\cdot\left(\vec{E}\times\vec{H}\right)$$

$$\vec{E}\cdot\vec{J} = -\mu\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} - \nabla\cdot\left(\vec{E}\times\vec{H}\right) - \varepsilon\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\vec{E}\cdot\vec{J} = -\frac{\mu}{2}\frac{\partial\vec{H}^2}{\partial t} - \frac{\varepsilon}{2}\frac{\partial\vec{E}^2}{\partial t} - \nabla\cdot\left(\vec{E}\times\vec{H}\right)$$

$$\frac{dW}{dt} = -\int\frac{1}{2}\left(\mu\frac{\partial\vec{H}^2}{\partial t} + \varepsilon\frac{\partial\vec{E}^2}{\partial t}\right)dV - \int\nabla\cdot\left(\vec{E}\times\vec{H}\right)dV$$

$$\frac{dW}{dt} = -\int\frac{1}{2}\left(\mu\frac{\partial\vec{H}^2}{\partial t} + \varepsilon_o\frac{\partial\vec{E}^2}{\partial t}\right)dV - \boxed{\oint\left(\vec{E}\times\vec{H}\right)}d\vec{a}$$

$$\frac{dW}{dt} \equiv -\frac{\partial\bar{U}_{EM}}{\partial t} - \oint\vec{S}\cdot d\vec{a}$$

Poynting Vector ($\vec{S}$)

Power flow out of the enclosed surface ($\oint\vec{S}\cdot d\vec{a}$)

EM power stored in the volume ($\frac{\partial\bar{U}_{EM}}{\partial t}$)

The decrease of the EM energy stored is to do work on a charge or due to the net energy escape through the surface.

Poynting Vector

$$\vec{S} \equiv \vec{E} \times \vec{H}$$

Magnitude gives power intensity = power/area
Direction gives direction of propagation

$$\vec{S}_{ave} \equiv \frac{1}{2}\Re e[\vec{E} \times \vec{H}^*]$$

Time average

$$S_{ave} = \frac{|E_o|^2}{2\eta} = \frac{\eta}{2}|H_o|^2$$

This is similar to P = ½ (V^2/Z) = ½ ($I^2 Z$)

Example

If solar illumination is characterized by a power density of 1 kW/m^2 at Earth's surface, find (a) the total power radiated by the sun,
(b) the total power intercepted by Earth, and
(c) the electric field of the power density incident upon Earth's surface, assuming that all the solar illumination is at a single frequency. The radius of Earth's orbit around the sun, R_s, is approximately 1.5 x 10^8 km, and Earth's mean radius R_E is 6380 km.

(a) Average power radiated $P_{sun} = S_{ave}(4\pi R_s^2) = (10^3)(4\pi)(1.5 \times 10^{11})^2 = 2.8 \times 10^{26}$ W

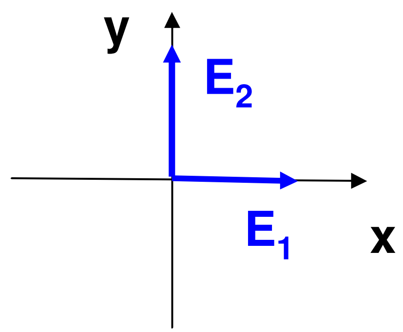
(b) Average power intercepted $P_E = S_{ave}(\pi R_E^2) = (10^3)(\pi)(6.38 \times 10^6)^2 = 1.28 \times 10^{17}$ W

(c) Electric field intensity …

$$S_{ave} = \frac{|E_o|^2}{2\eta}$$

$$E_o = \sqrt{2\eta_o S_{ave}} = \sqrt{2(377)(10^3)} = 870 \quad \text{V/m}$$

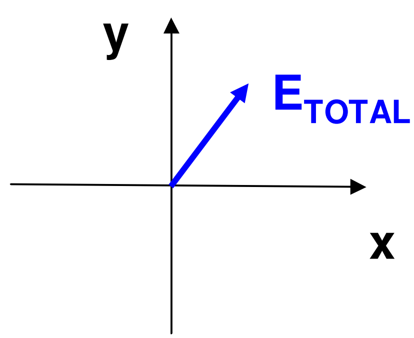
Linear Polarization



Defined by the direction of E-field. For example:

$$\vec{E}_1(\vec{r},t) = \hat{x}E_{o1}\cos(\omega t - \beta z)$$ Horizontal polarization

$$\vec{E}_2(\vec{r},t) = \hat{y}E_{o2}\cos(\omega t - \beta z)$$ Vertical polarization



A linear combination of these….

$$\vec{E}_{TOTAL} = \vec{E}_1 + \vec{E}_2$$ is still linearly polarized

If $E_{o1} = E_{o2}$, then E_{TOTAL} is at 45 degrees.

Any EM wave can be expressed as a linear combination of horizontal & vertical polarized field.

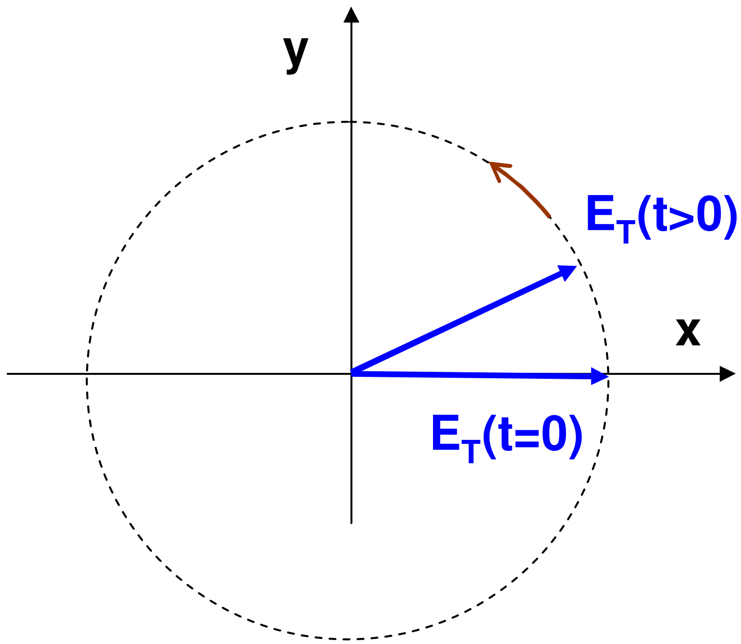
Circular Polarization

If **E**$_1$ & **E**$_2$ are 90 degrees out-of-phase, & $E_{o1} = E_{o2}$:

$$\vec{E}_1(\vec{r},t) = \hat{x}E_{o1}\cos(\omega t - \beta z)$$ Horizontal polarization

$$\vec{E}_2(\vec{r},t) = \hat{y}E_{o2}\sin(\omega t - \beta z)$$ Vertical polarization

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2$$

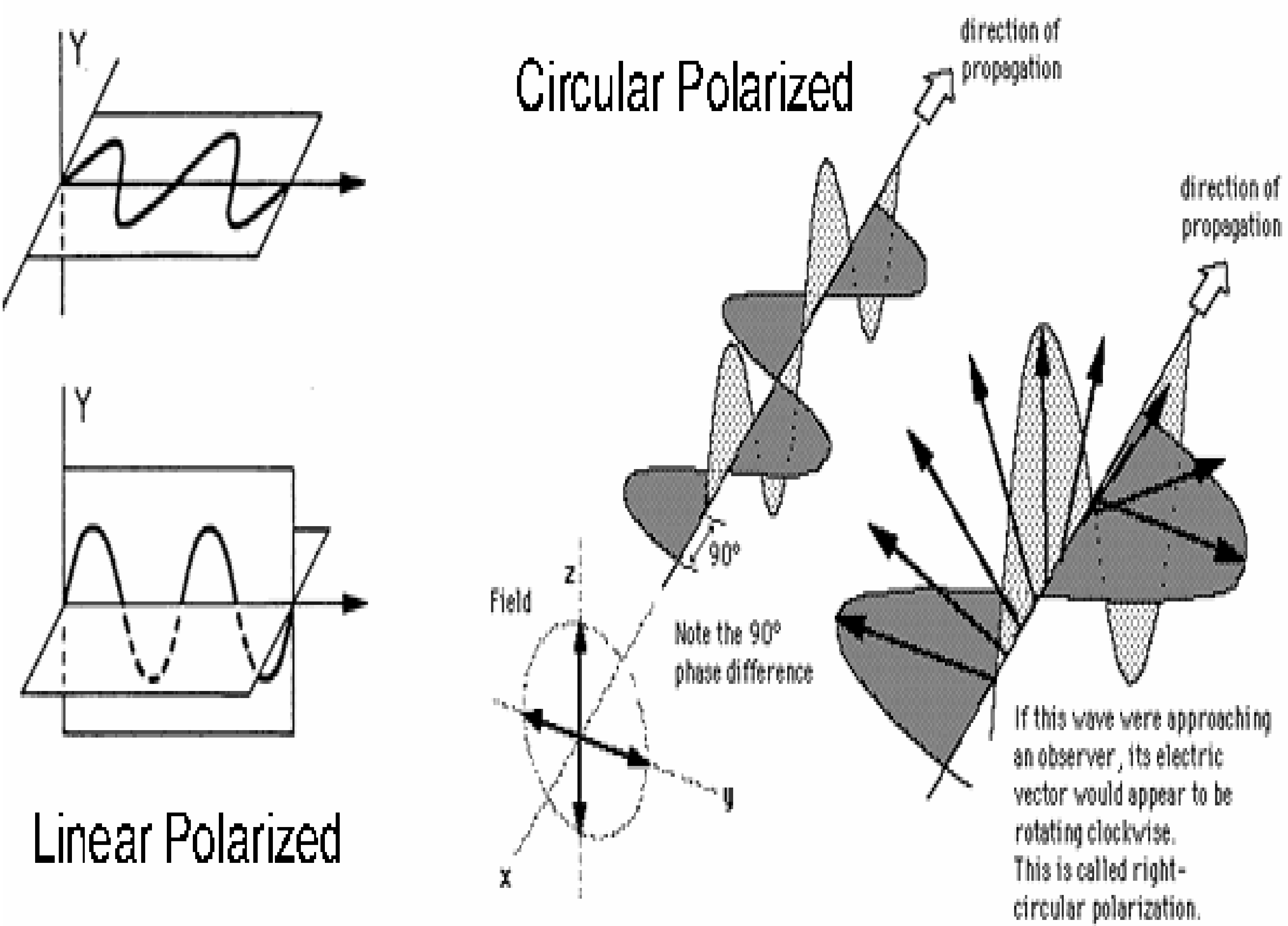


RHCP
Right-Hand Circularly Polarized

$$\vec{E}(\vec{r},t) = \hat{x}E_o\cos(\omega t - \beta z) + \hat{y}E_o\sin(\omega t - \beta z)$$

$$\vec{E}(\vec{r},t) = \hat{x}E_o e^{j(\omega t - \beta z)} + \hat{y}E_o e^{j(\omega t - \beta z - \pi/2)}$$

$$\vec{E}(\vec{r},t) = (\hat{x} - j\hat{y})E_o e^{j(\omega t - \beta z)}$$

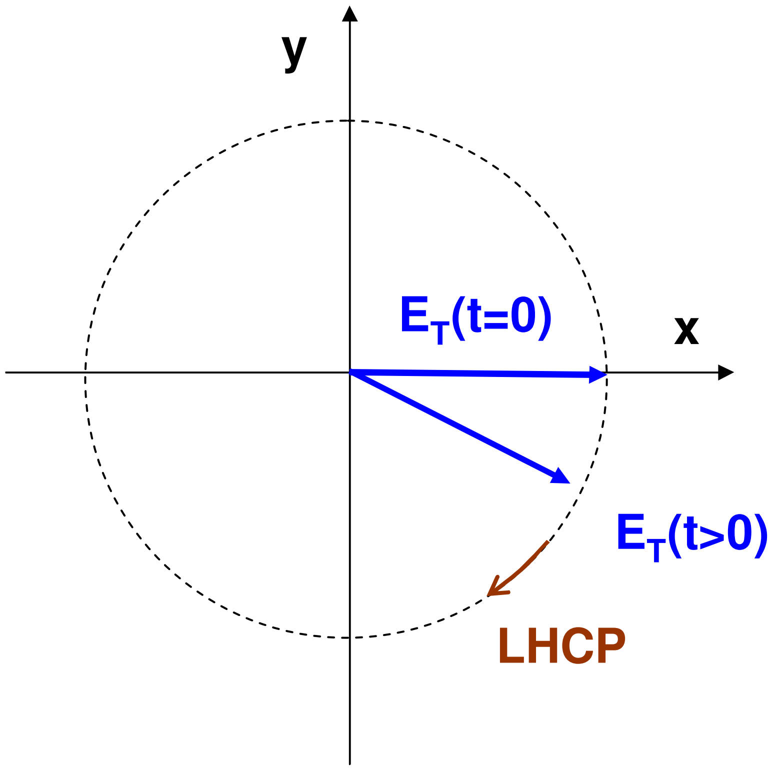

Y
Y
Linear Polarized
Circular Polarized
direction of propagation
direction of propagation
90°
z
Field
Note the 90° phase difference
y
x
If this wave were approaching an observer, its electric vector would appear to be rotating clockwise.
This is called right-circular polarization.

LHCP

$$\vec{E}(\vec{r},t) = \hat{x}E_o\cos(\omega t - \beta z) - \hat{y}E_o\sin(\omega t - \beta z)$$

$$\vec{E}(\vec{r},t) = \hat{x}E_o e^{j(\omega t - \beta z)} + \hat{y}E_o e^{j(\omega t - \beta z + \pi/2)}$$

$$\vec{E}(\vec{r},t) = (\hat{x} + j\hat{y})E_o e^{j(\omega t - \beta z)}$$



Circular polarization in nature

Rose Chafer (a beetle) & others…

Reflection from its surface is LHCP., due to the helical molecular structure of their shells.

Mantis Shrimp (Stomatopod Crustacean)

Able to sense LHCP & RHCP, over a wide spectrum of color, better than man-made instrument.

Most complex eye structure in animal kingdom.

Fastest animal (striking) & produces over 200 lbs of striking force !!!

Nature Photonics 3, 641-644 (2009)
Roberts, Chiou, Marshall & Cronin,

Sheila Patek at TED.com

a popular dish known as
"pissing shrimp" (攋尿蝦)

Circular polarization in starlight

Most starlight are <u>slightly</u> circularly polarized.

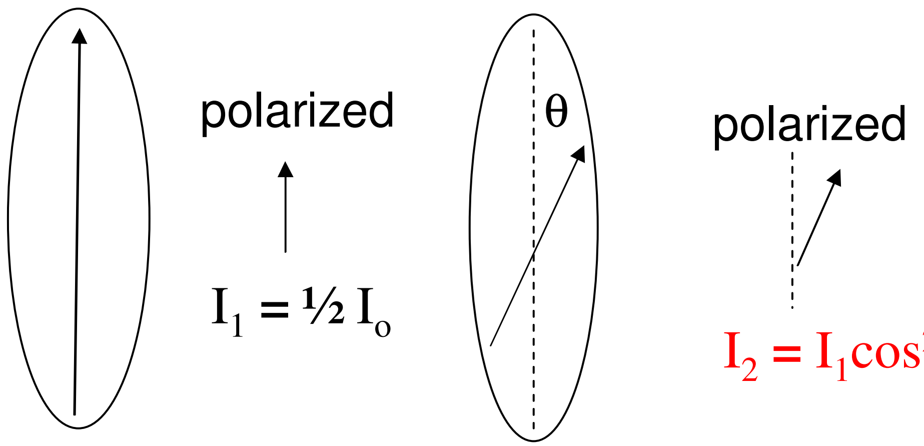
The interstellar medium (low density dust) can produce circularly polarized (CP) light from unpolarized light by sequential scattering from elongated interstellar grains (with complex index of refraction) aligned in different directions. One possibility is twisted grain alignment along the line of sight due to variation in the galactic magnetic field; another is the line of sight passes through multiple clouds.

Polarizer

unpolarized

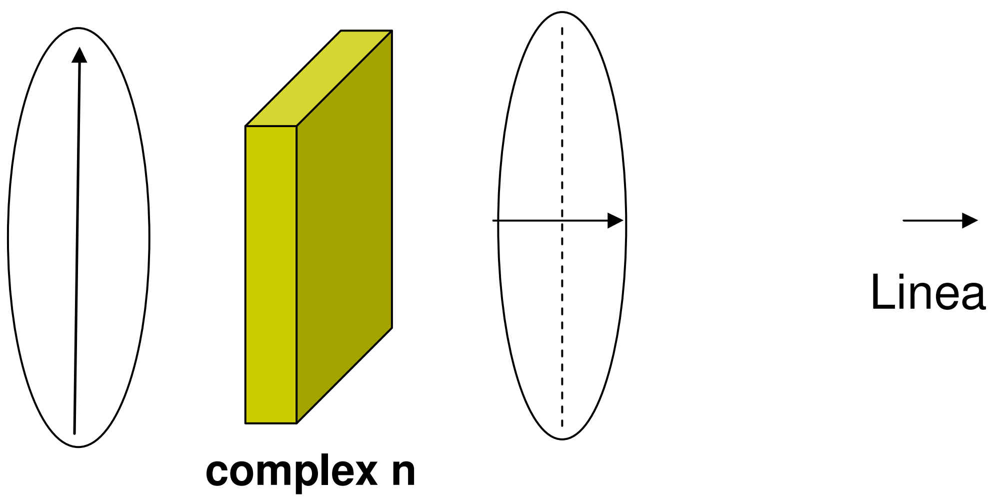
I_o

intensity



$I_1 = ½\, I_o$

$I_2 = I_1\cos^2\theta = ½\, I_o\cos^2\theta$



Linearly polarized

Elliptical Polarization

If $E_{o1} \neq E_{o2}$

$$\vec{E}(\vec{r},t) = \hat{x}E_{o1}\cos(\omega t - \beta z) \pm \hat{y}E_{o2}\sin(\omega t - \beta z)$$

$$\vec{E}(\vec{r},t) = (\hat{x}E_{o1} \mp j\hat{y}E_{o2})e^{j(\omega t - \beta z)}$$

RHEP
LHEP

Or, if the phase difference is not 90 degrees

$$\vec{E}(\vec{r},t) = \hat{x}E_o e^{j(\omega t - \beta z)} + \hat{y}E_o e^{j(\omega t - \beta z + \delta)}$$

$$\vec{E}(\vec{r},t) = (\hat{x} + \hat{y}e^{j\delta})E_o e^{j(\omega t - \beta z)}$$

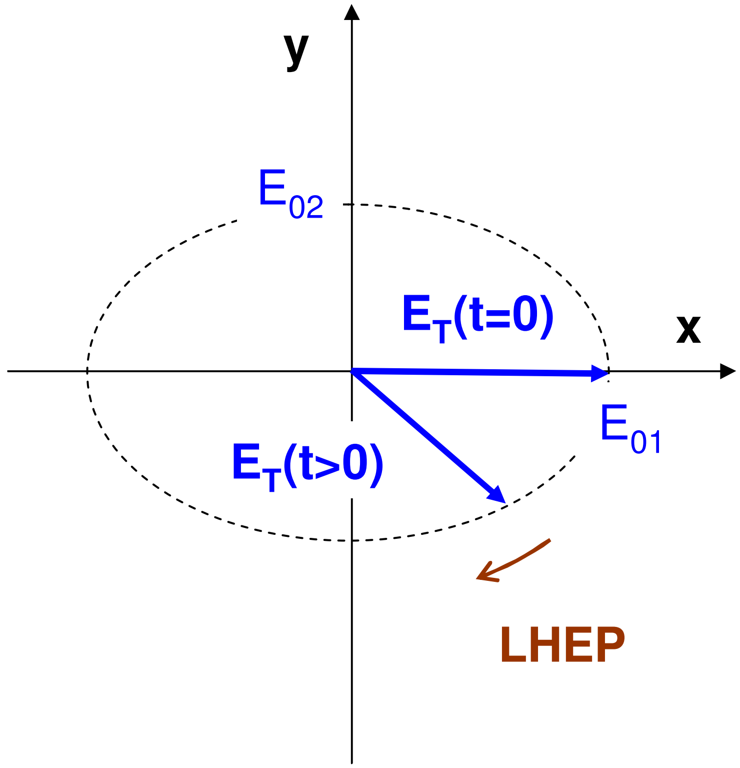
RHEP $(\delta < 0)$

LHEP $(\delta > 0)$

Special cases:

if $\delta = -\pi/2$, $\rightarrow$ RHCP

if $\delta = \pi/2$, $\rightarrow$ LHCP



Polarization Generalization

$$\vec{E}(\vec{r},t) = \hat{x}a_x e^{j(\omega t-\beta z)} + \hat{y}a_y e^{j(\omega t-\beta z+\delta)}$$

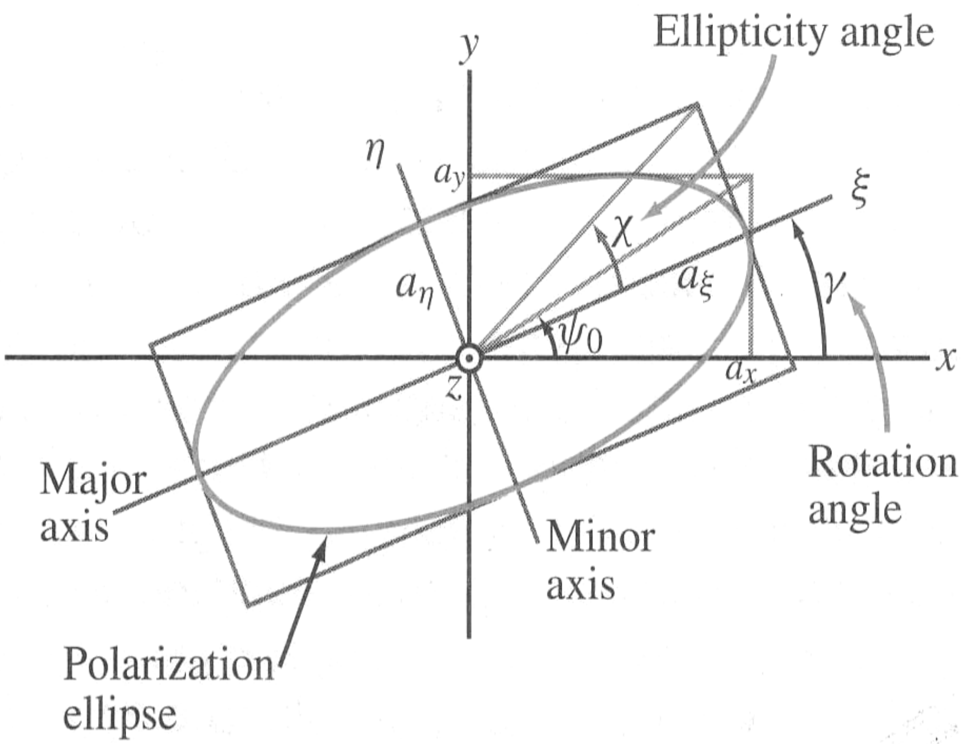


Figure : Polarization ellipse in the x–y plane, with the wave traveling in the z-direction (out of the page).

$$R \equiv \frac{a_\xi}{a_\eta}$$

axial ratio ≥ 1
= 1 circular
= ∞ linear

$$\tan\chi \equiv \pm\frac{1}{R}$$

ellipticity angle
$\chi > 0$ when $\sin\delta > 0 \rightarrow$ LH
$\chi < 0$ when $\sin\delta < 0 \rightarrow$ RH

$$\tan\psi_o \equiv \frac{a_y}{a_x}$$

auxiliary angle
$0 \le \psi_o \le \pi/2$

$$\sin 2\chi = \sin(2\psi_o)\sin\delta$$

$-\pi/4 \le \chi \le \pi/4$

$$\tan 2\gamma = \tan(2\psi_o)\cos\delta$$

rotation angle $-\pi/2 \le \gamma \le \pi/2$

$\gamma > 0$ if $\cos\delta > 0$

$\gamma < 0$ if $\cos\delta < 0$

Polarization States

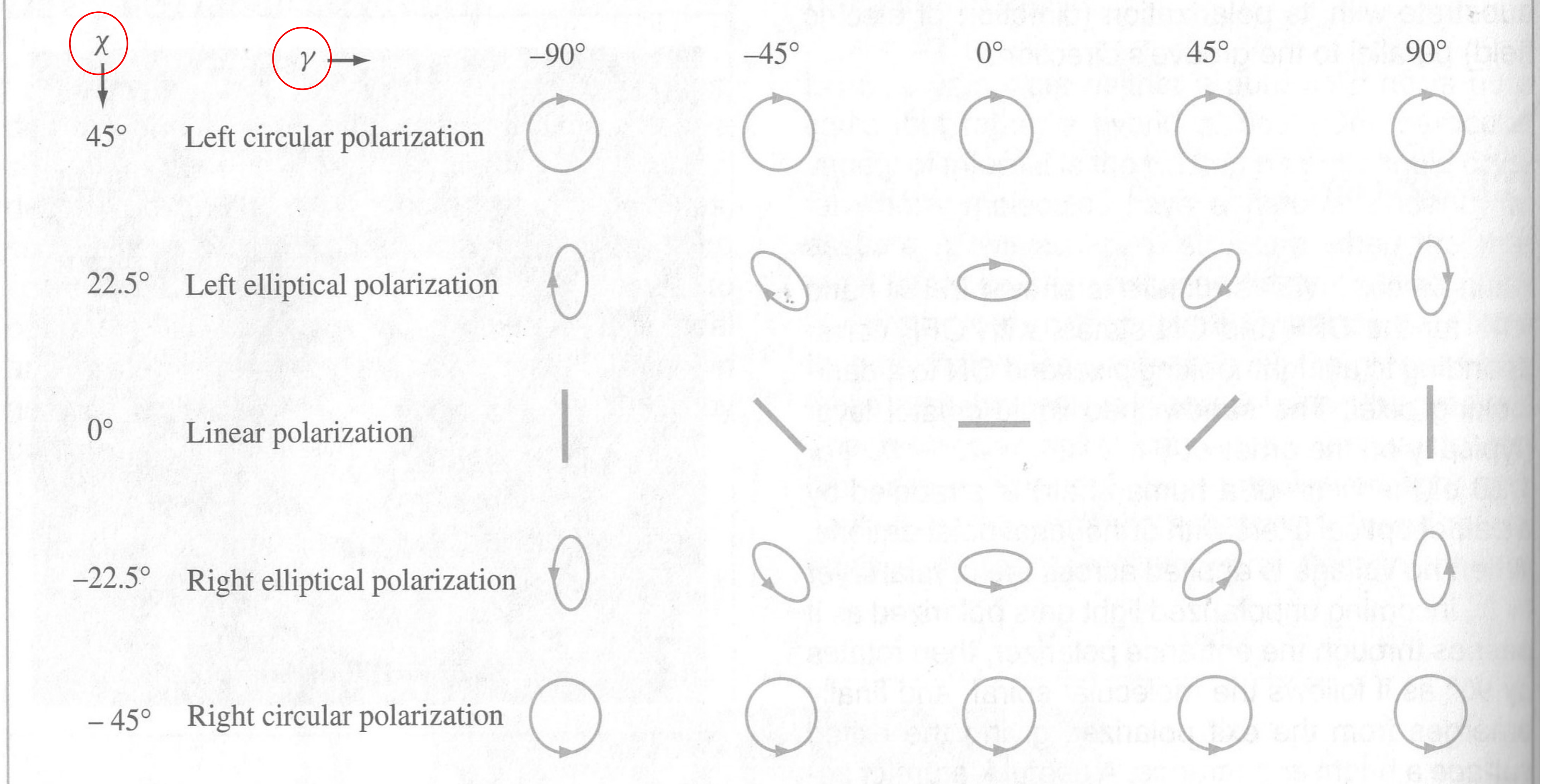


Figure Polarization states for various combinations of the polarization angles (γ, χ) for a wave traveling out of the page.

e.g. what's the polarization?

$$\vec{E}(z,t) = \hat{x}3\cos(\omega t - kz + \pi/6) - \hat{y}4\sin(\omega t - kz + \pi/4)$$

$$\vec{E}(z,t) = \hat{x}3e^{j(\omega t - kz + \pi/6)} - \hat{y}4e^{j(\omega t - kz + \pi/4 - \pi/2)}$$

$$\vec{E}(z,t) = \hat{x}3e^{j(\omega t - kz + \pi/6)} + \hat{y}4e^{j(\omega t - kz + \pi/4 - \pi/2 + \pi)}$$

$$\vec{E}(z,t) = \hat{x}3e^{j(\omega t - kz + \pi/6)} + \hat{y}4e^{j(\omega t - kz + 3\pi/4)}$$

$$\vec{E}(z,t) = \left(\hat{x}3 + \hat{y}4e^{j7\pi/12}\right)e^{j(\omega t - kz + \pi/6)}$$

$$\vec{E}(z,t) = \left(\hat{x}3 + \hat{y}4e^{j105^o}\right)e^{j(\omega t - kz + \pi/6)}$$

$$\delta = 105^o$$

$$\sin\delta = \sin(105^o) = 0.966 > 0 \Rightarrow \chi > 0 \Rightarrow LH$$

$$\cos\delta = \cos(105^o) = -0.256 < 0 \Rightarrow \gamma < 0$$

$$\tan\psi_o \equiv \frac{a_y}{a_x} = \frac{4}{3}$$

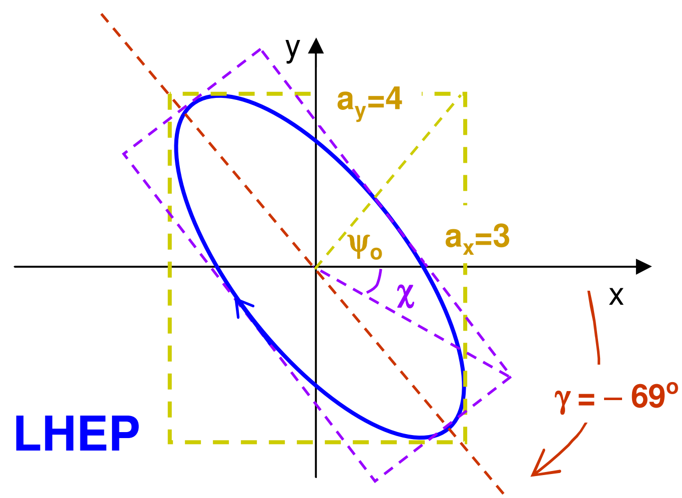
$$\psi_o = 53^o$$

$$\sin 2\chi = \sin(2\psi_o)\sin\delta = \sin(106^o)\sin(105^o)$$

$$\chi = 34^o$$

$$\tan 2\gamma = \tan(2\psi_o)\cos\delta = \tan(106^o)\cos(105^o)$$

$$\gamma = -69^o$$



Linear combination

Any LP EM wave can be written as linear combination of RHCP + LHCP

e.g. $\vec{E}(\vec{r},t) = \hat{x}E_o \cos(\omega t - \beta z)$

$$\vec{E}_{RHCP}(\vec{r},t) = (\hat{x} - j\hat{y})E_1 e^{j(\omega t - \beta z)}$$

$$\vec{E}_{LHCP}(\vec{r},t) = (\hat{x} + j\hat{y})E_2 e^{j(\omega t - \beta z)}$$

$$\vec{E}_T(\vec{r},t) = \vec{E}_{RHCP}(\vec{r},t) + \vec{E}_{LHCP}(\vec{r},t)$$

$$\vec{E}_T(\vec{r},t) = \left[\hat{x}(E_1 + E_2) + j\hat{y}(E_2 - E_1)\right]e^{j(\omega t - \beta z)}$$

$$E_2 = E_1 = \frac{E_o}{2}$$

More generally, any EM wave can be written as linear combination of RHEP + LHEP

Exercise

The amplitudes of an elliptically polarized plane wave traveling in a lossless, nonmagnetic medium with $\varepsilon_r = 4$ and $H_{xo} = 8$ mA/m, and $H_{yo} = 6$ mA/m. Determine the average power flowing through an aperture in the y-z plane if its area is 20 m^2.