Statically Indeterminate Structures
Force Method Example

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Steps in Solving an Indeterminate Structure using the Force Method

Determine degree of Indeterminacy
Let *n* =degree of indeterminacy
(i.e. the structure is indeterminate to the nth degree)

Chapter 3

Define Primary Structure and the *n* Redundants

Define the Primary Problem

Solve for the *n* Relevant Deflections in Primary Problem

Chapters 3,4,5 then 7 or 8

Define the *n* Redundant Problems

Solve for the *n* Relevant Deflections in each Redundant Problem

Chapters 3,4,5 then 7 or 8

Write the *n* Compatibility Equations at Relevant Points

Solve the *n* Compatibility Equations to find the *n* Redundants

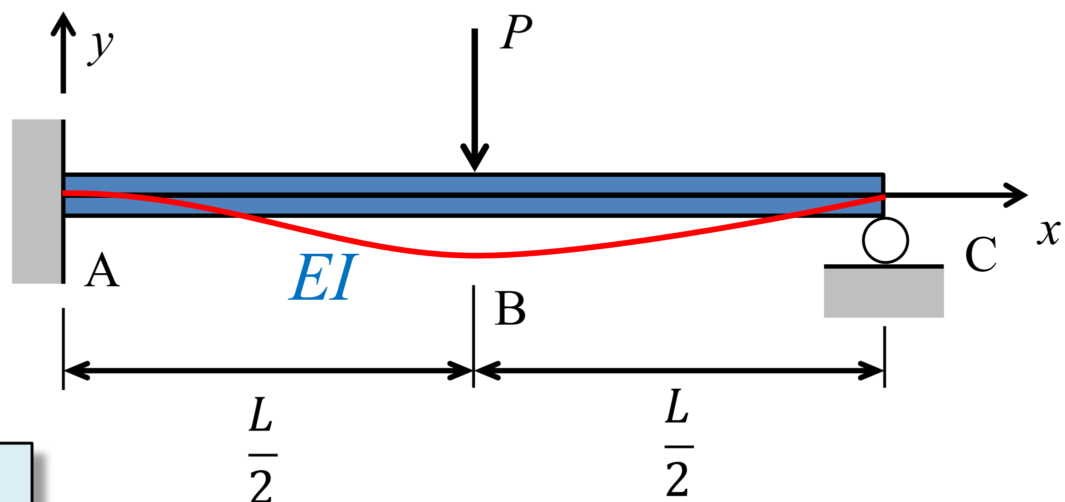
Use the Equations of Equilibrium to solve for the remaining unknowns

Chapter 3

Construct Internal Force Diagrams (if necessary)

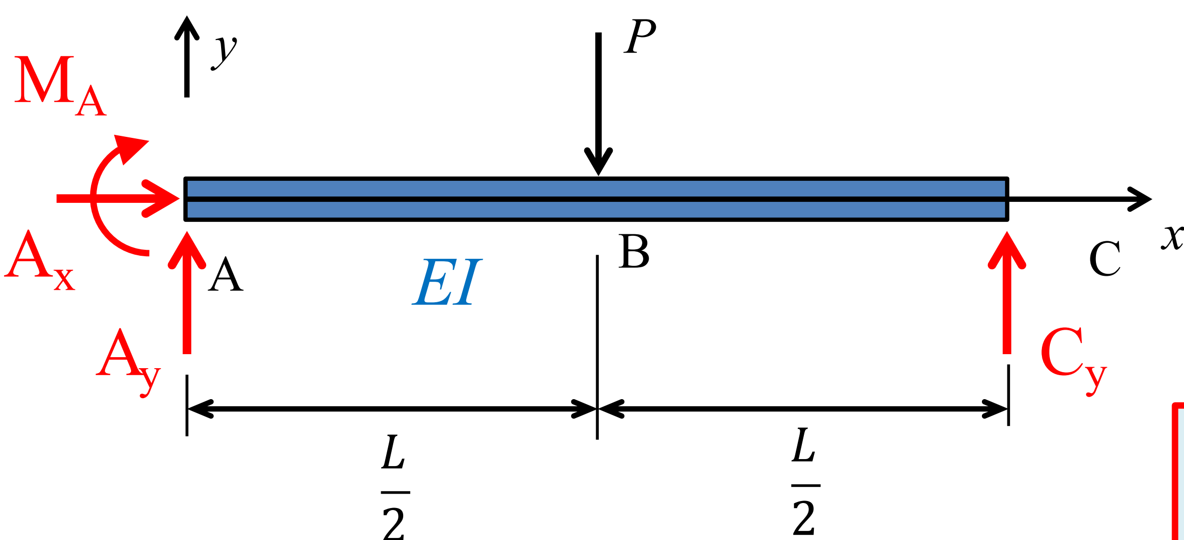
Chapters 3,4,5

Example Problem



For the indeterminate beam subject to the point load, *P*, find the support reactions at A and C. *EI* is constant.

FBD



Beam is stable

$X = 4$

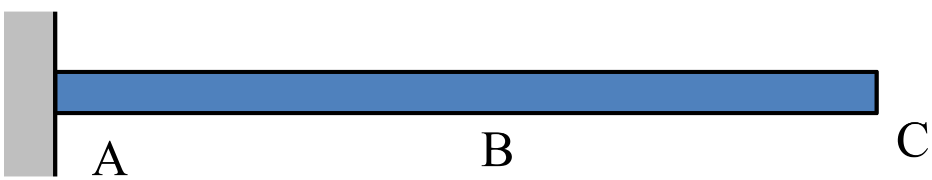
$3n = 3(1) = 3$

Statically Indeterminate to the 1st degree

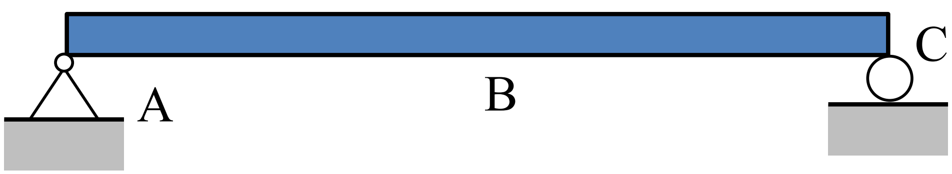
Define Primary Structure and Redundant

- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.

Primary Structure | **Redundant**



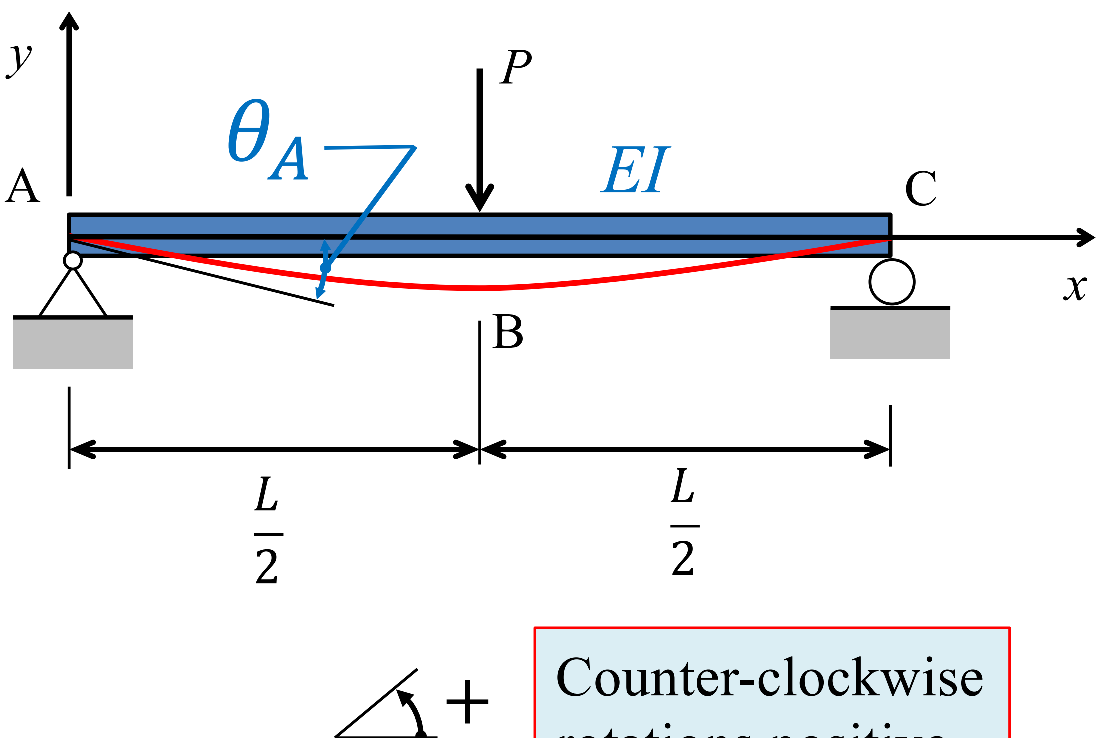
C_y



M_A

Define and Solve the Primary Problem

- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.



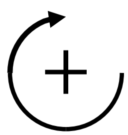
From Tabulated Solutions

Counter-clockwise rotations positive

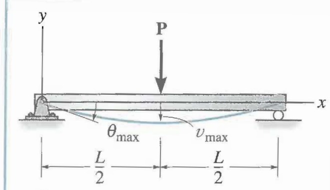
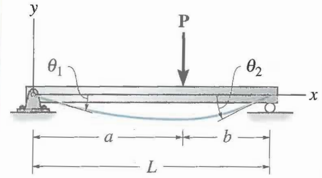
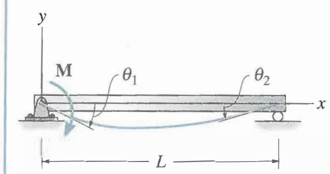
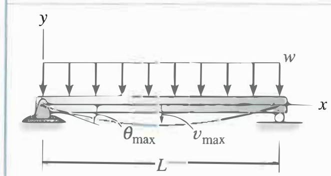
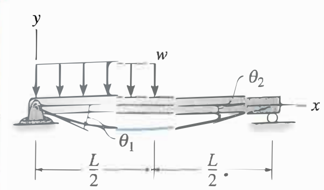
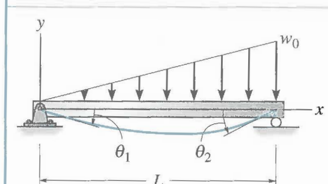
$$\theta_A = -\frac{PL^2}{16EI}$$

Tabulated Solutions

v + ↑

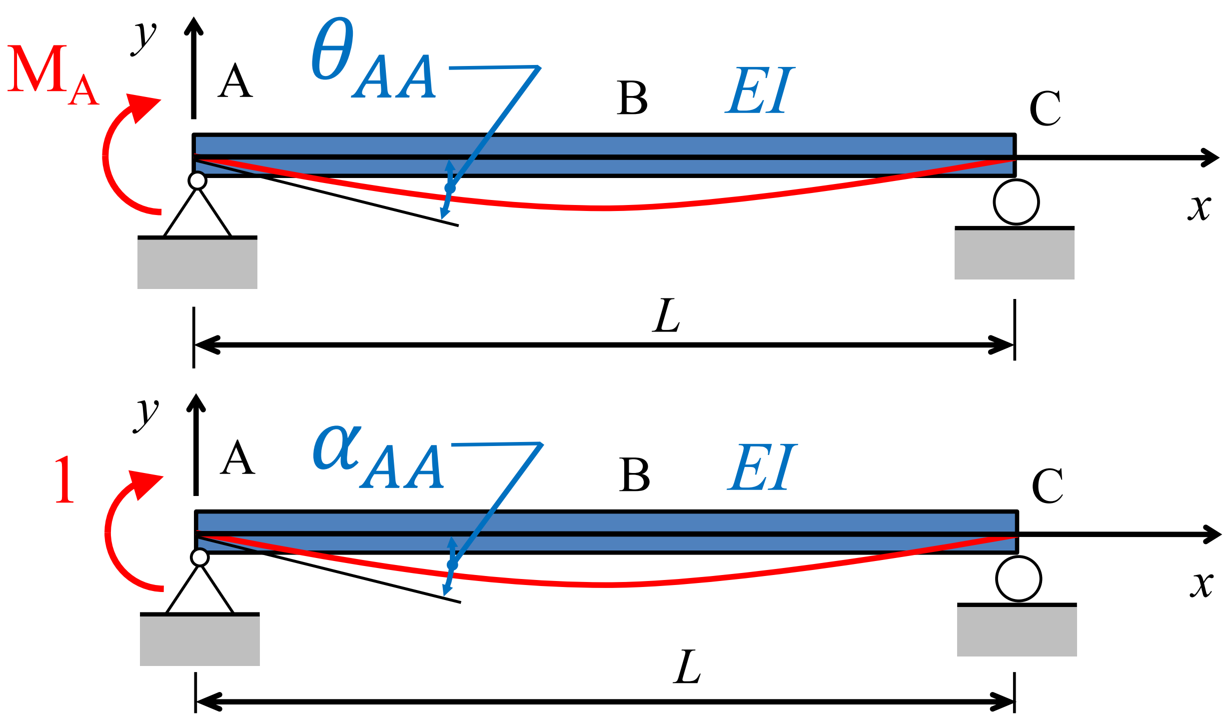
θ 

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{Pab(L+b)}{6EIL}$ $\theta_2 = \frac{-Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(-x^2 - b^2 + L^2)$ $0 \le x \le a$
	$\theta_1 = \frac{ML}{3EI}$ $\theta_2 = \frac{-ML}{6EI}$	$v_{max} = \frac{-ML^2}{\sqrt{243}\,EI}$	$v = \frac{-Mx}{6LEI}(x^2 - 3Lx + 2L^2)$ $0 \le x \le L$
	$\theta_{max} = \frac{wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$ $0 \le x \le L$
	$\theta_1 = \frac{3wL^3}{128EI}$ $\theta_2 = \frac{-7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \frac{-5wL^4}{768EI}$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$\theta_1 = \frac{7w_0L^3}{360EI}$ $\theta_2 = \frac{-w_0L^3}{45EI}$		$v = \frac{-w_0x}{360LEI}(3x^4 - 10L^2x^2 + 7L^4)$ $0 \le x \le L$

Define and Solve the Redundant Problem

- There are the same number of redundant problems as degrees of indeterminacy;
- Define a reference coordinate system;
- Apply only one redundant to the primary structure;
- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.
- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;



Redundant Problem

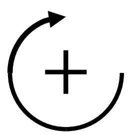
$$\theta_{AA} = M_A \alpha_{AA}$$

From Tabulated Solutions

$$\alpha_{AA} = -\frac{L}{3EI}$$

Tabulated Solutions

v + ↑

θ 

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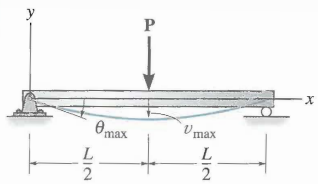
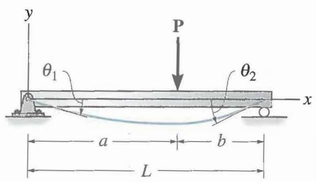
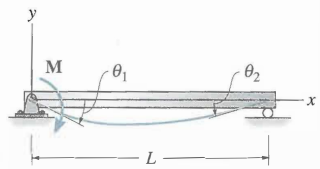
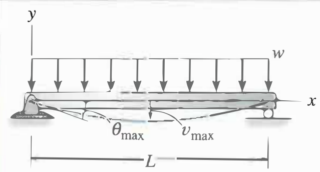
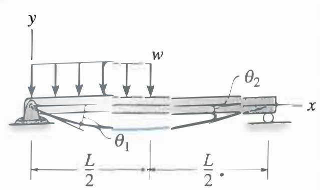
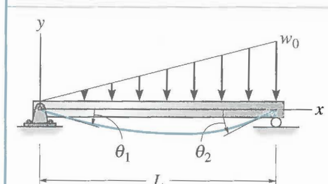
	$\theta_{max} = \frac{PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{Pab(L+b)}{6EIL}$ $\theta_2 = \frac{-Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \frac{-Pba}{6EIL}$ $\cdot(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(-x^2 - b^2 + L^2)$ $0 \le x \le a$
	$\theta_1 = \frac{ML}{3EI}$ $\theta_2 = \frac{-ML}{6EI}$	$v_{max} = \frac{-ML^2}{\sqrt{243}\,EI}$	$v = \frac{-Mx}{6LEI}(x^2 - 3Lx + 2L^2)$ $0 \le x \le L$
	$\theta_{max} = \frac{wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$ $0 \le x \le L$
	$\theta_1 = \frac{3wL^3}{128EI}$ $\theta_2 = \frac{-7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \frac{-5wL^4}{768EI}$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$\theta_1 = \frac{7w_0L^3}{360EI}$ $\theta_2 = \frac{-w_0L^3}{45EI}$		$v = \frac{-w_0x}{360LEI}(3x^4 - 10L^2x^2 + 7L^4)$ $0 \le x \le L$

Compatibility Equation at Point A

Compatibility at Point A

$$\theta_A + \theta_{AA} = 0$$

Compatibility Equation in terms of Redundant and Flexibility Coefficient

$$\theta_A + M_A \alpha_{AA} = 0$$

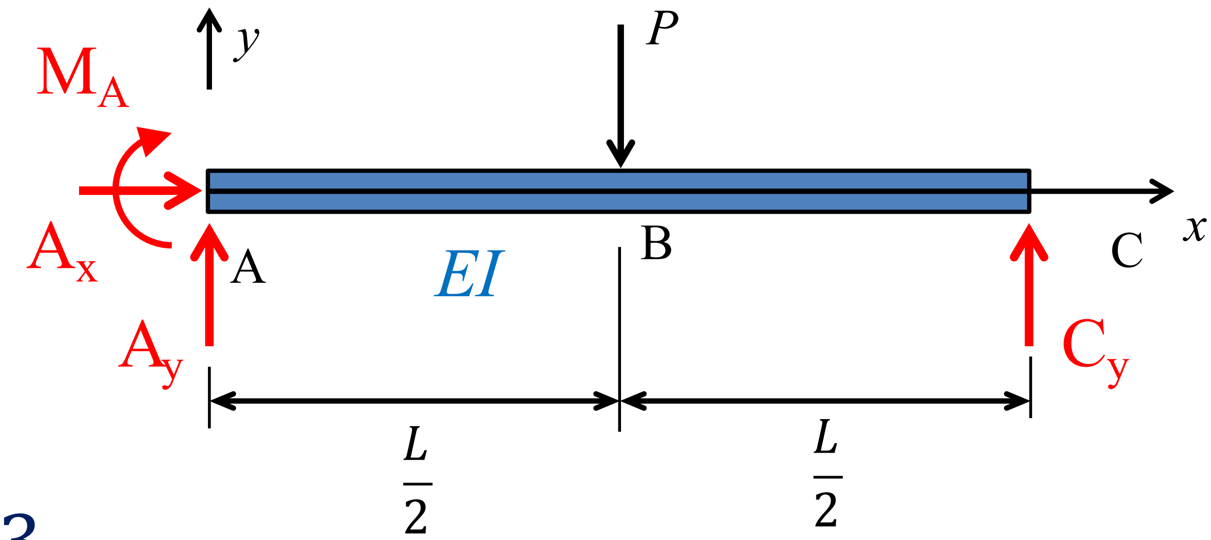
$$-\frac{PL^2}{16EI} + M_A\left(-\frac{L}{3EI}\right) = 0$$

Solve for M_A

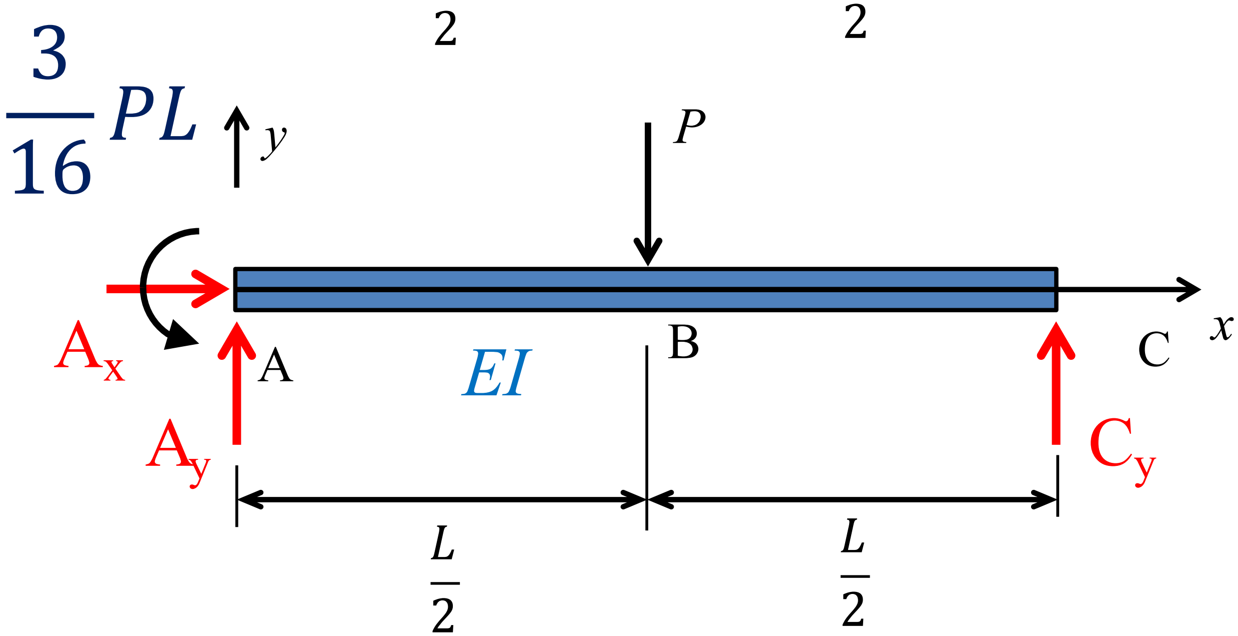
$$M_A = \frac{PL^2}{16EI}\left(-\frac{3EI}{L}\right)$$

$$M_A = -\frac{3}{16}PL$$

Free Body Diagram

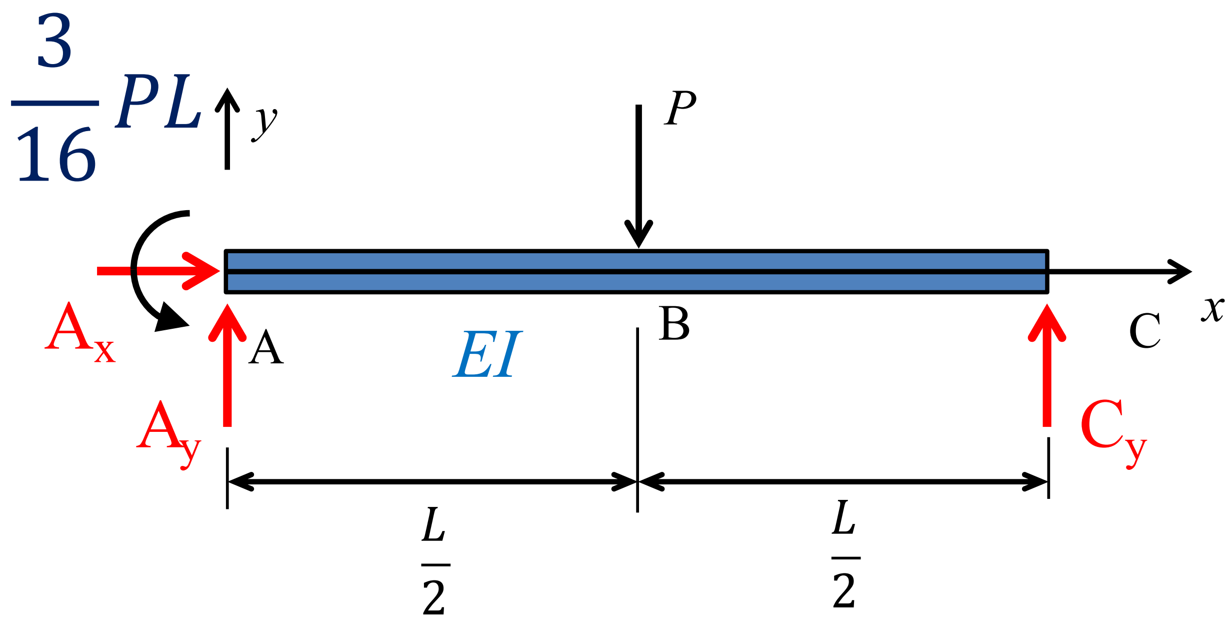


$$M_A = -\frac{3}{16}PL$$



Can now use equilibrium equations to find the remaining three unknowns

Find Remaining Unknowns



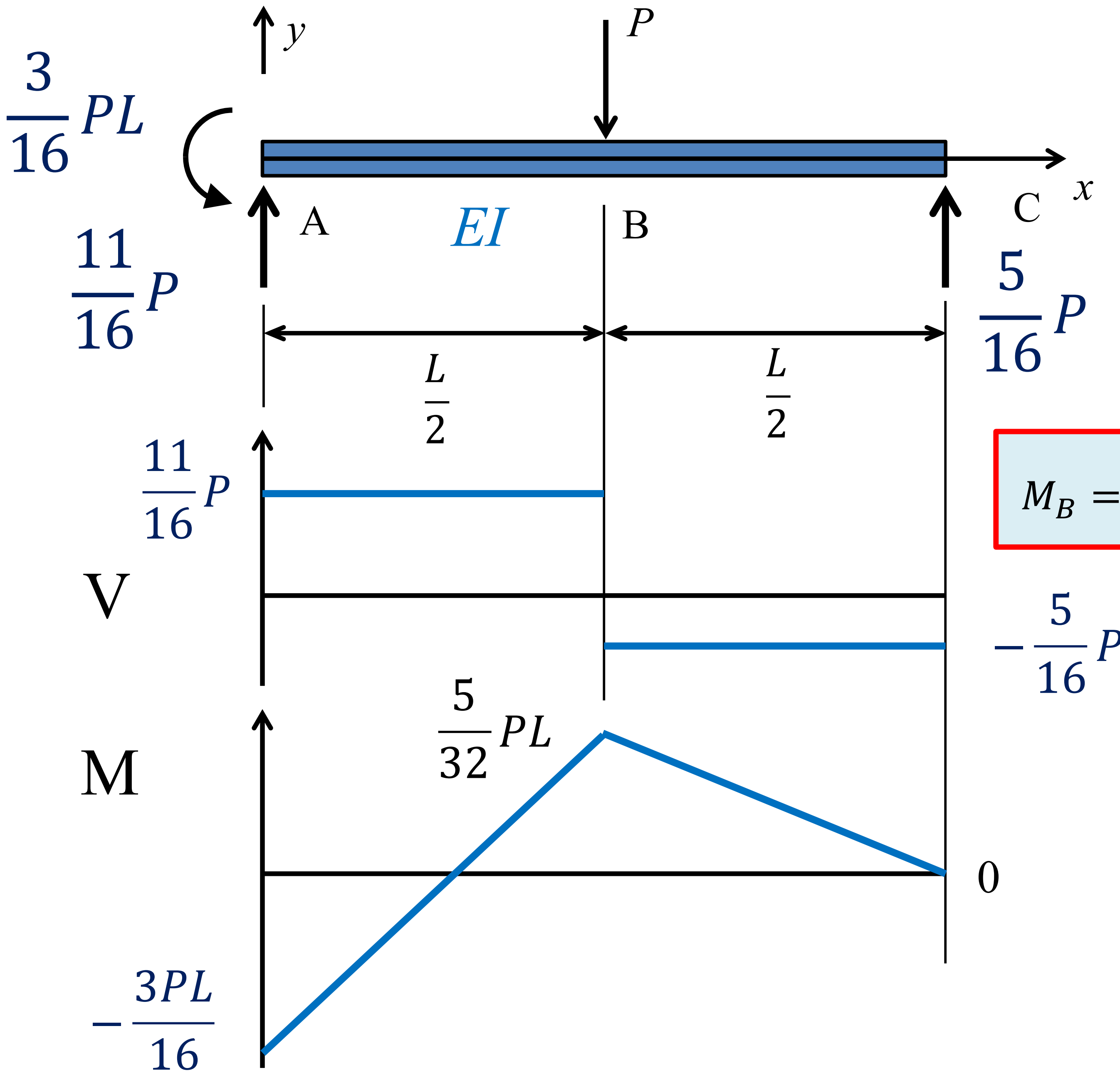
Can now use equilibrium equations to find the remaining three unknowns

$$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$$

$$\circlearrowleft + \sum M_A = 0 \rightarrow C_y = \frac{5}{16}P$$

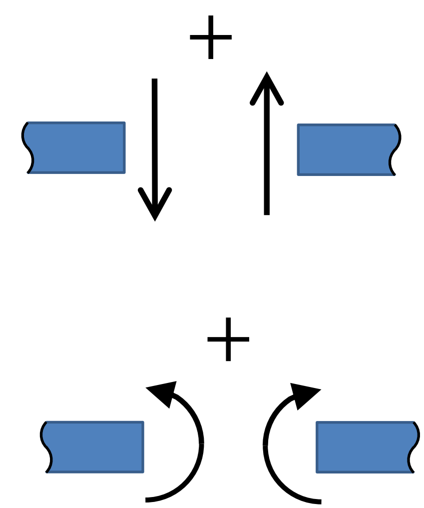
$$+\uparrow \sum F_y = 0 \rightarrow A_y = \frac{11}{16}P$$

Draw V and M Diagrams of the Beam



$$M_B - M_A = \left(\frac{11}{16}P\right)\left(\frac{L}{2}\right)$$

$$M_B = -\frac{3}{16}PL + \frac{11}{32}PL = \frac{5}{32}PL$$



Superposition of Primary and Redundant Problems