Constructing Beam Influence Lines for Internal Forces

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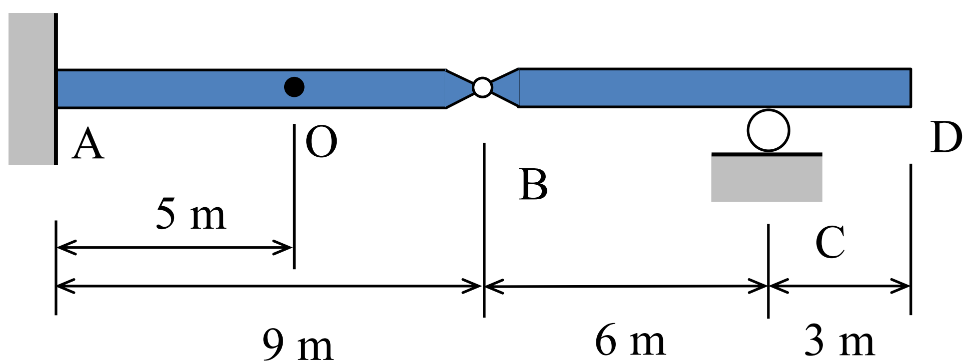
Construction of Influence Lines

The overhanging beam shown has a fixed support at A, a roller support at C and an internal hinge at B. Construct influence lines for:

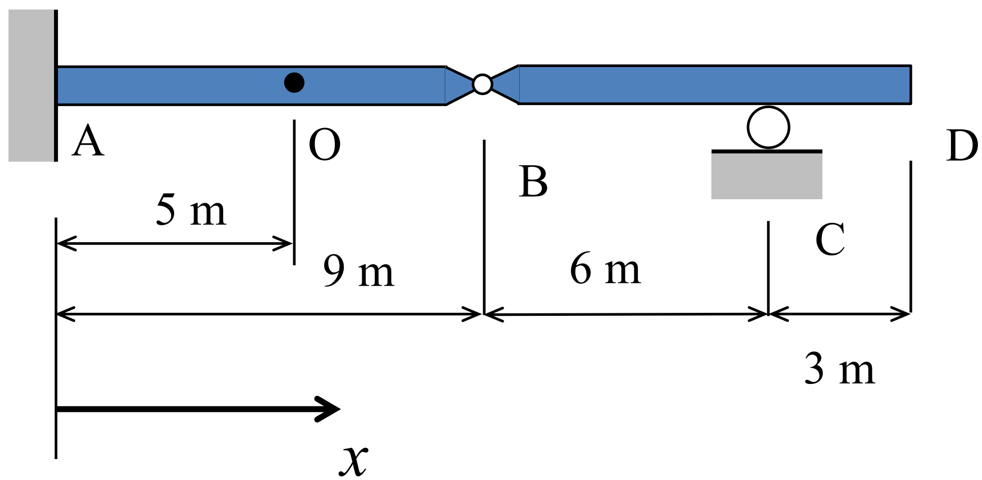
1. The internal shear force at point O:
2. The internal bending moment at point O.



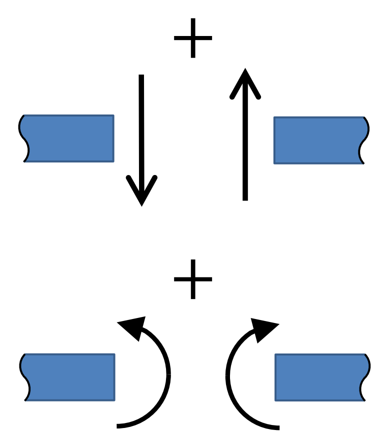
Choose Reference Coordinate and Sign Convention

The overhanging beam shown has a fixed support at A, a roller support at C and an internal hinge at B. Construct influence lines for:

1. The internal shear force at point O:
2. The internal bending moment at point O.



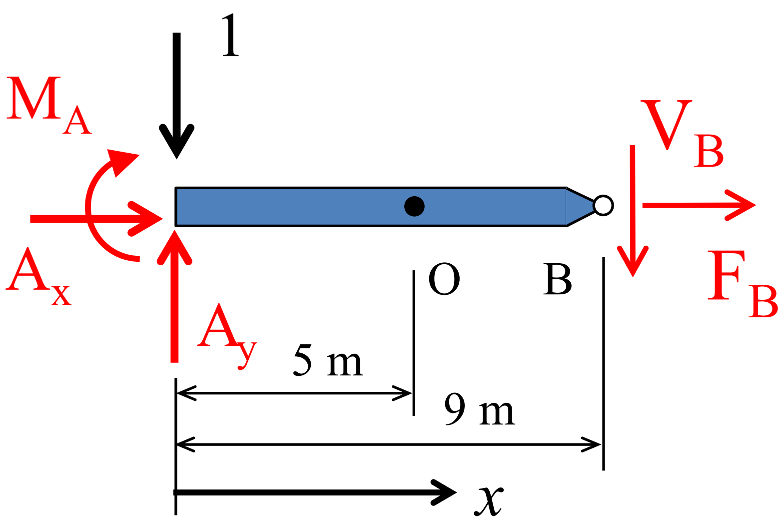
Sign Convention for Positive Internal Forces

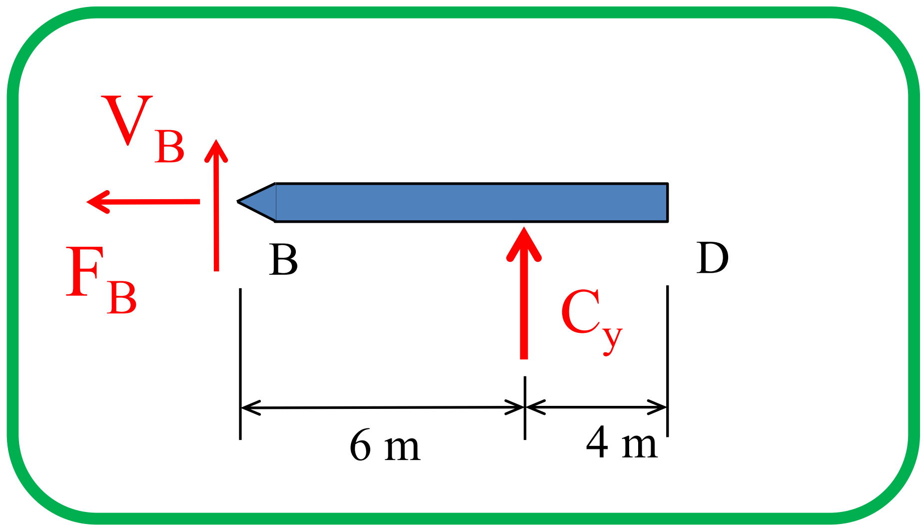


Place Unit Load at $x = 0$ (Point A)

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium





$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = 0$

$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

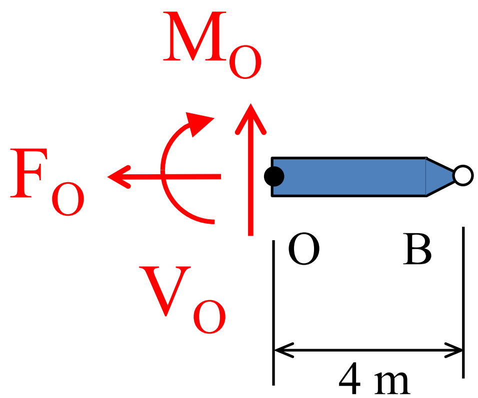
$+\uparrow \sum F_y = 0 \rightarrow A_y = 1$

$\circlearrowleft + \sum M_B = 0 \rightarrow C_y = 0$

$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$

$+\uparrow \sum F_y = 0 \rightarrow V_B = 0$

FBD of Segment OB for Unit Load at $x = 0$



$$V_B = 0$$

$$F_B = 0$$

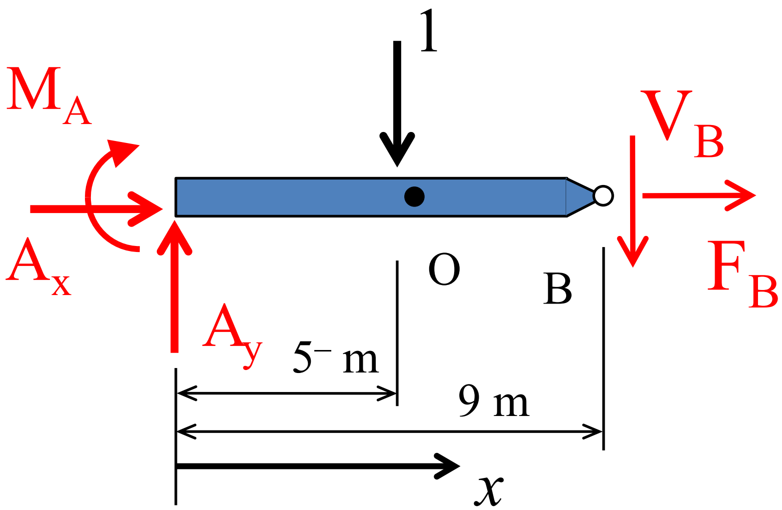
$$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = 0}$$

$$\xrightarrow{+} \sum F_x = 0 \rightarrow F_O = 0$$

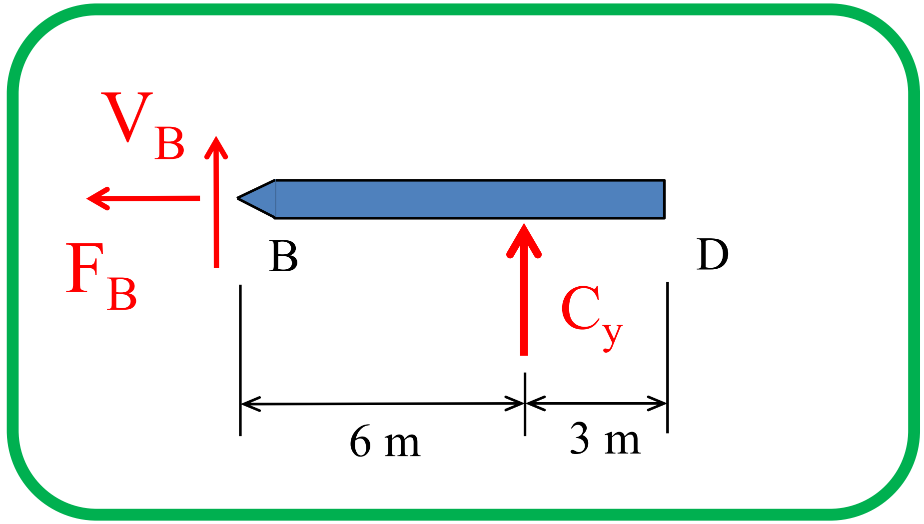
$$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = 0}$$

Place Unit Load at $x = 5^{-}$ m (Just to the Left of Point O)

Free-body Diagrams



6 Unknowns – 6 Equations of Equilibrium

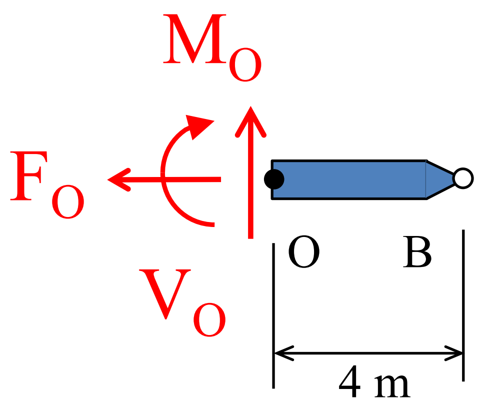


$$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = -5 \text{ m} \qquad \circlearrowleft + \sum M_B = 0 \rightarrow C_y = 0$$

$$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0 \qquad \xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow A_y = 1 \qquad +\uparrow \sum F_y = 0 \rightarrow V_B = 0$$

FBD of Segment OB for Unit Load at $x = 5^{-}$ m (Just to the Left of Point O)



$$V_B = 0$$

$$F_B = 0$$

$$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = 0}$$

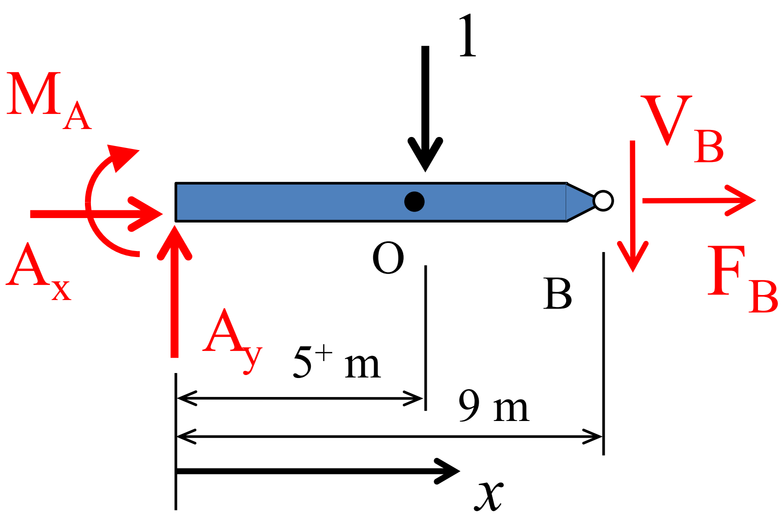
$$\xrightarrow{+} \sum F_x = 0 \rightarrow F_O = 0$$

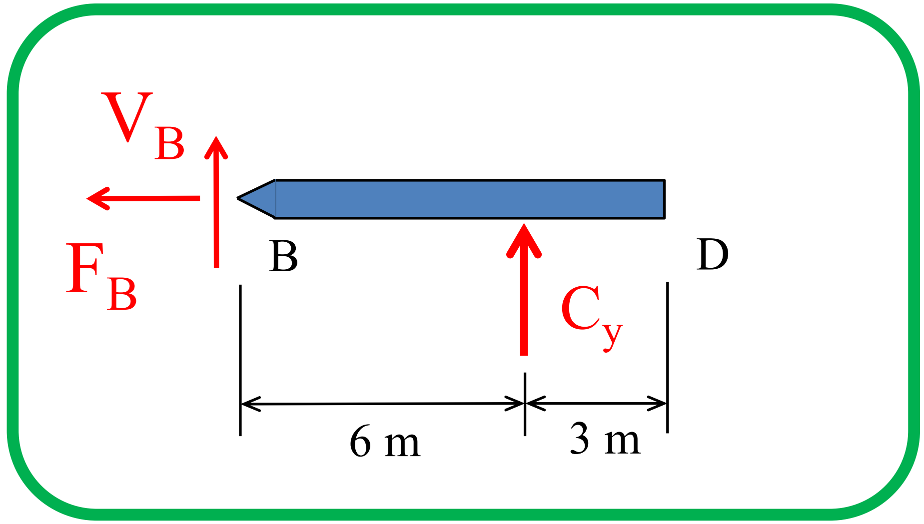
$$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = 0}$$

Place Unit Load at $x = 5^+$ m (Just to the Right of Point O)

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium





$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = -5$ m

$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

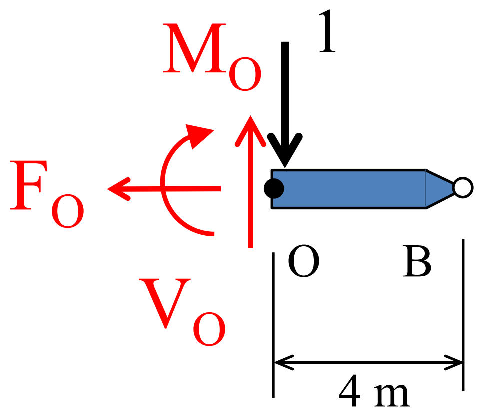
$+\uparrow \sum F_y = 0 \rightarrow A_y = 1$

$\circlearrowleft + \sum M_B = 0 \rightarrow C_y = 0$

$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$

$+\uparrow \sum F_y = 0 \rightarrow V_B = 0$

FBD of Segment OB for Unit Load at $x = 5^{+}$ m (Just to the Right of Point O)



$$V_B = 0$$

$$F_B = 0$$

$$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = 0}$$

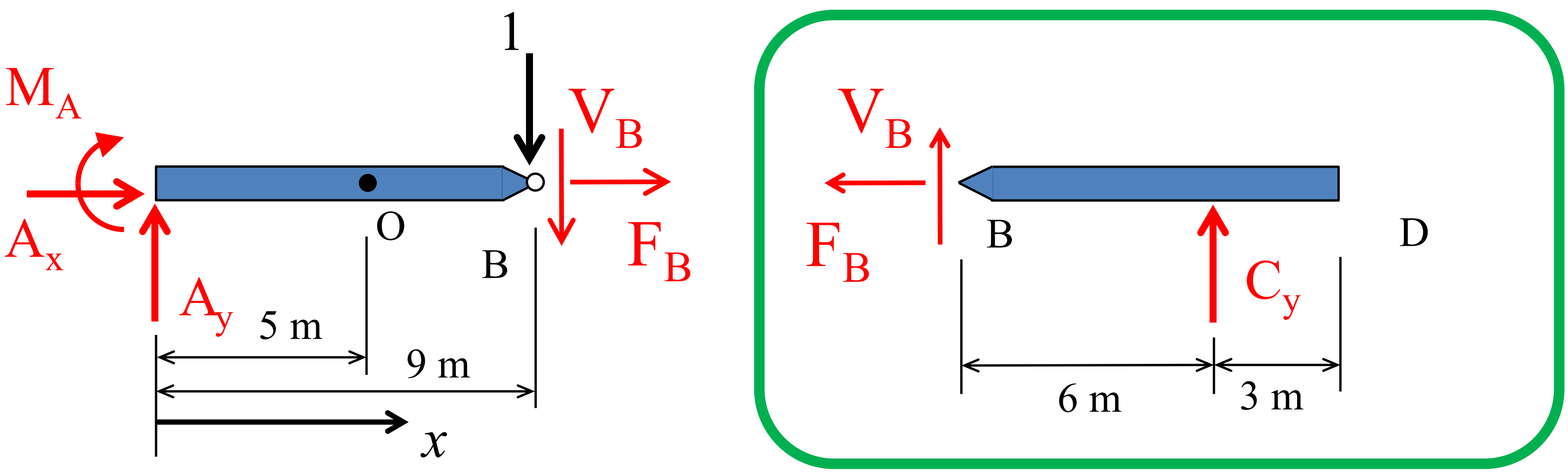
$$\xrightarrow{+} \sum F_x = 0 \rightarrow F_O = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = 1}$$

Place Unit Load at $x = 9^{-}$ m

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium



$$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = -9 \text{ m}$$

$$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$$

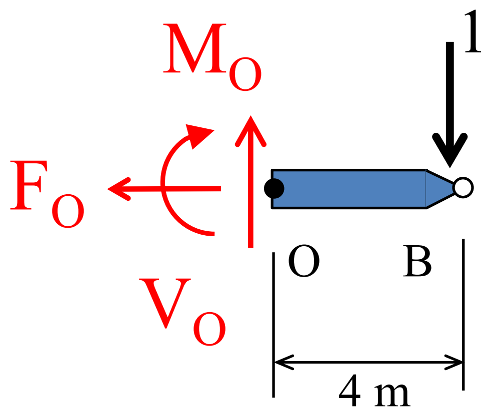
$$+\uparrow \sum F_y = 0 \rightarrow A_y = 1$$

$$\circlearrowleft + \sum M_B = 0 \rightarrow C_y = 0$$

$$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow V_B = 0$$

FBD of Segment OB for Unit Load at $x = 9^{-}$ m



$V_B = 0$

$F_B = 0$

$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = -4\text{ m}}$

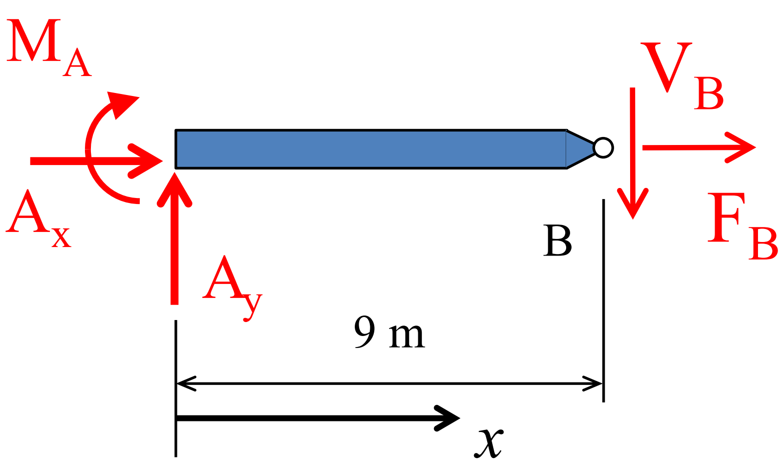
$\xrightarrow{+} \sum F_x = 0 \rightarrow F_O = 0$

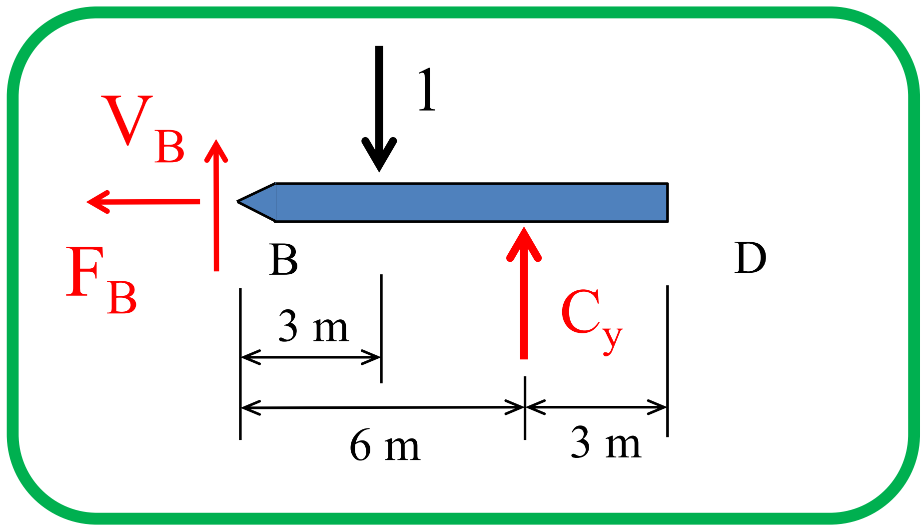
$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = 1}$

Place Unit Load at $x = 12$ m

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium





$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = -4.5$ m

$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

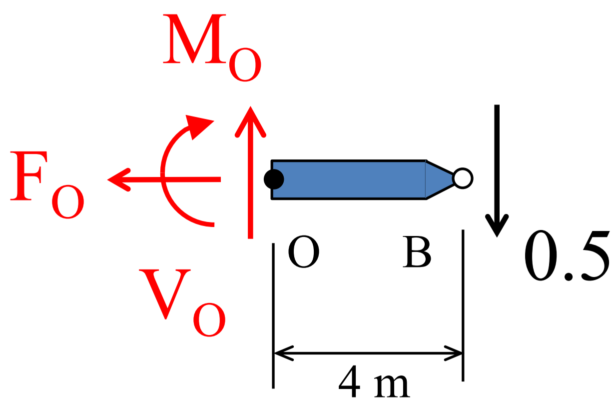
$+\uparrow \sum F_y = 0 \rightarrow A_y = 0.5$

$\circlearrowleft + \sum M_B = 0 \rightarrow C_y = 0.5$

$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$

$+\uparrow \sum F_y = 0 \rightarrow V_B = 0.5$

FBD of Segment OB for Unit Load at $x = 12$ m



$V_B = 0.5$

$F_B = 0$

$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = -2 \text{ m}}$

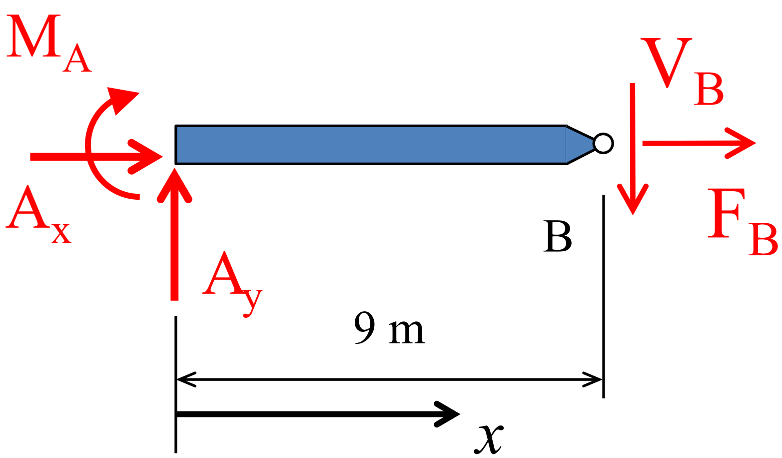
$\xrightarrow{+} \sum F_x = 0 \rightarrow F_O = 0$

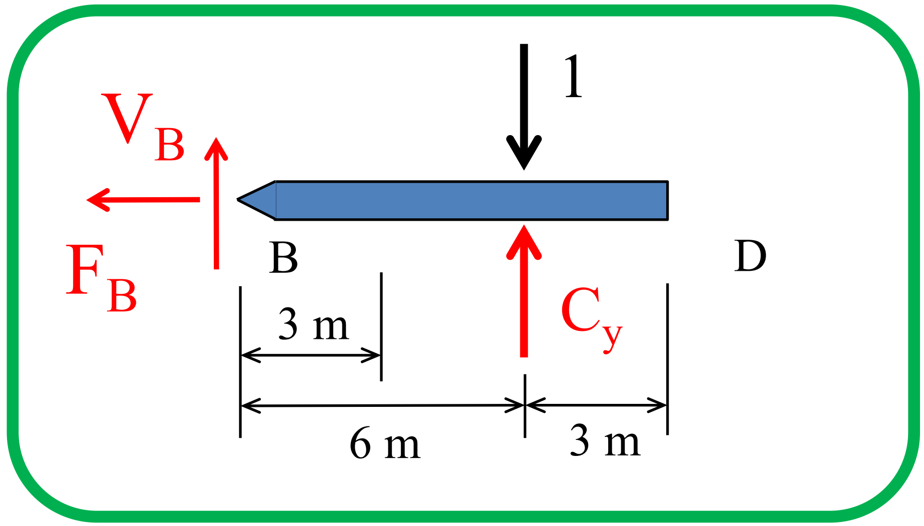
$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = 0.5}$

Place Unit Load at x = 15 m

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium





$\circlearrowleft + \sum M_A = 0 \;\rightarrow\; M_A = 0$

$\xrightarrow{+} \sum F_x = 0 \;\rightarrow\; A_x = 0$

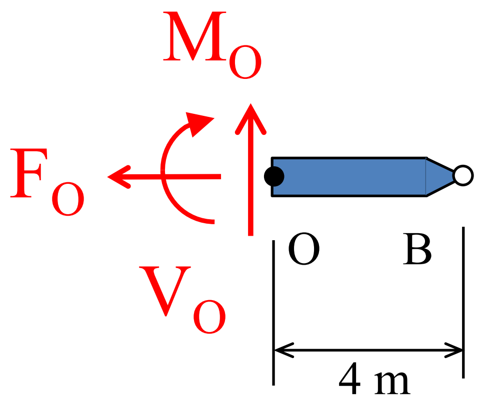
$+\uparrow \sum F_y = 0 \;\rightarrow\; A_y = 0$

$\circlearrowleft + \sum M_B = 0 \;\rightarrow\; C_y = 1$

$\xrightarrow{+} \sum F_x = 0 \;\rightarrow\; F_B = 0$

$+\uparrow \sum F_y = 0 \;\rightarrow\; V_B = 0$

FBD of Segment OB for Unit Load at $x = 15$ m



$$V_B = 0$$

$$F_B = 0$$

$$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = 0}$$

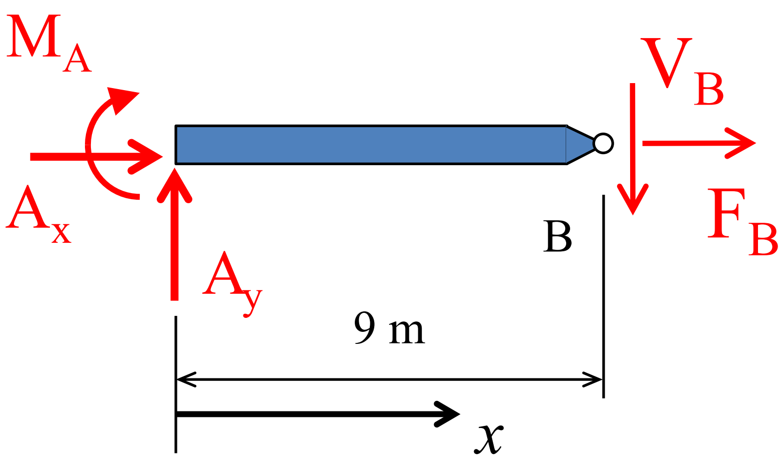
$$\overset{+}{\rightarrow} \sum F_x = 0 \rightarrow F_O = 0$$

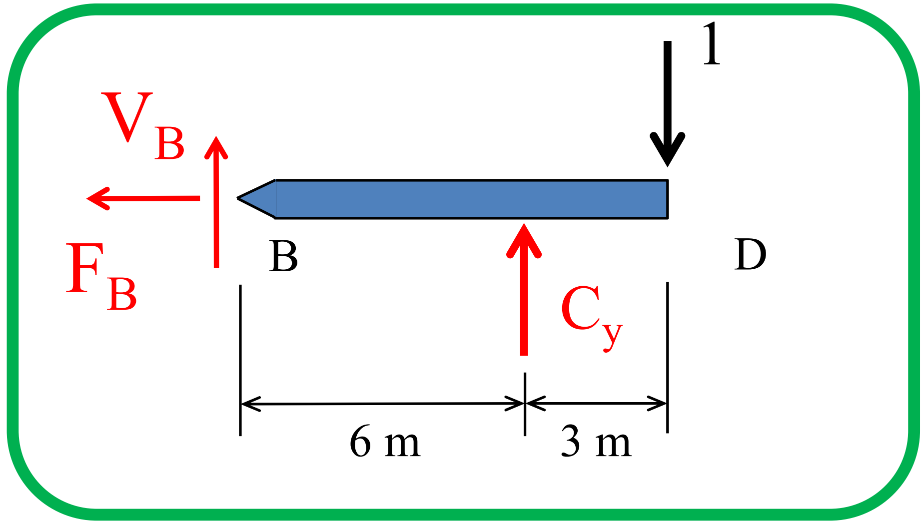
$$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = 0}$$

Place Unit Load at $x = 18$ m

Free-body Diagrams

6 Unknowns – 6 Equations of Equilibrium





$\circlearrowleft + \sum M_A = 0 \rightarrow M_A = 4.5$ m

$\circlearrowleft + \sum M_B = 0 \rightarrow C_y = 1.5$

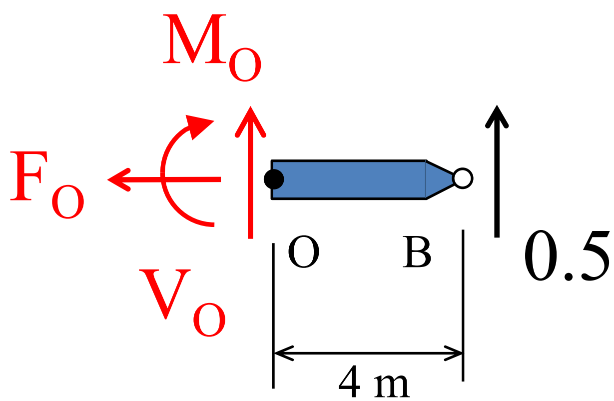
$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

$\xrightarrow{+} \sum F_x = 0 \rightarrow F_B = 0$

$+\uparrow \sum F_y = 0 \rightarrow A_y = -0.5$

$+\uparrow \sum F_y = 0 \rightarrow V_B = -0.5$

FBD of Segment OB for Unit Load at $x = 12$ m



$V_B = -0.5$

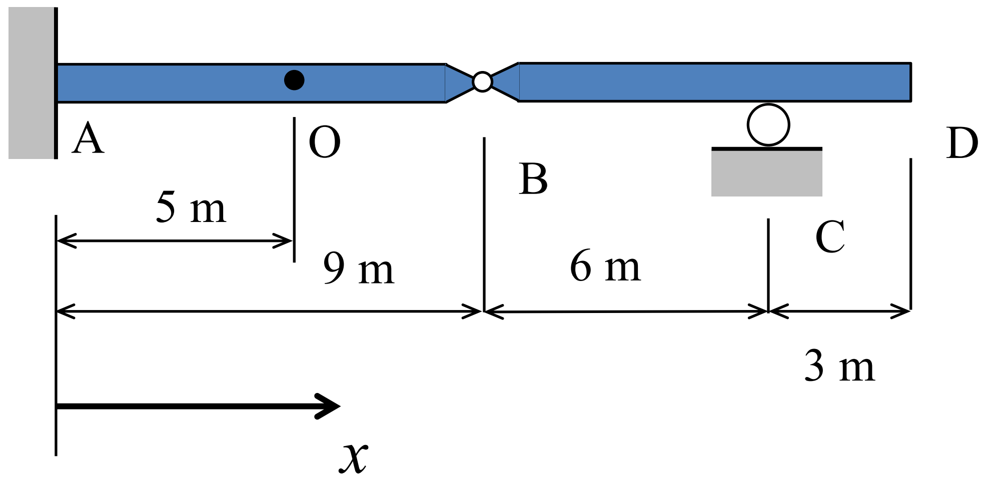
$F_B = 0$

$\circlearrowleft + \sum M_O = 0 \rightarrow \boxed{M_O = 2 \text{ m}}$

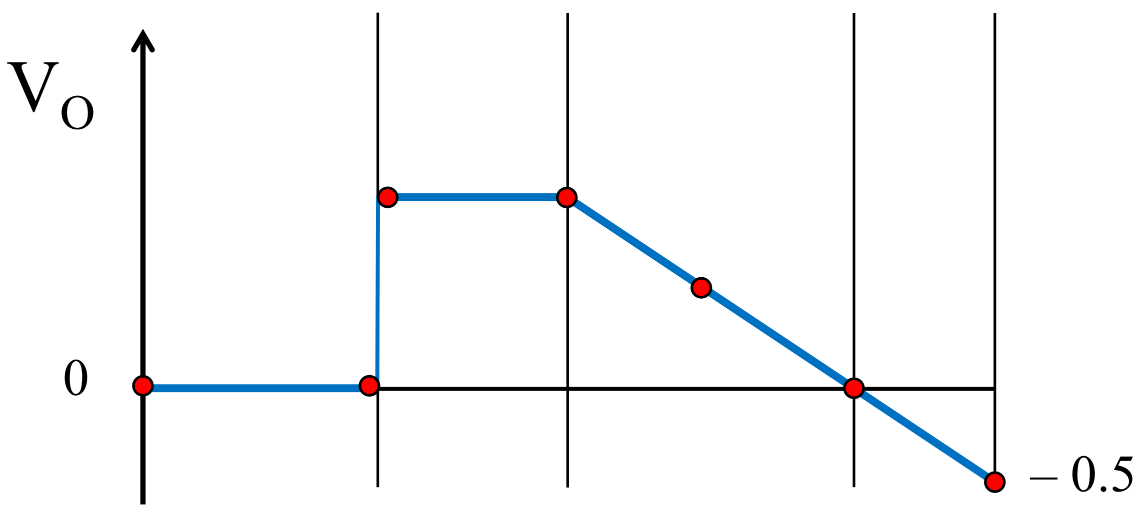
$\xrightarrow{+} \sum F_x = 0 \rightarrow F_O = 0$

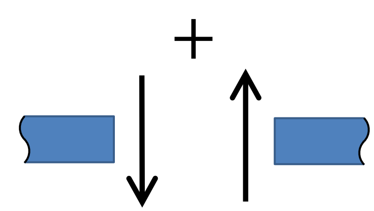
$+\uparrow \sum F_y = 0 \rightarrow \boxed{V_O = -0.5}$

Plot the Influence Line for V_O

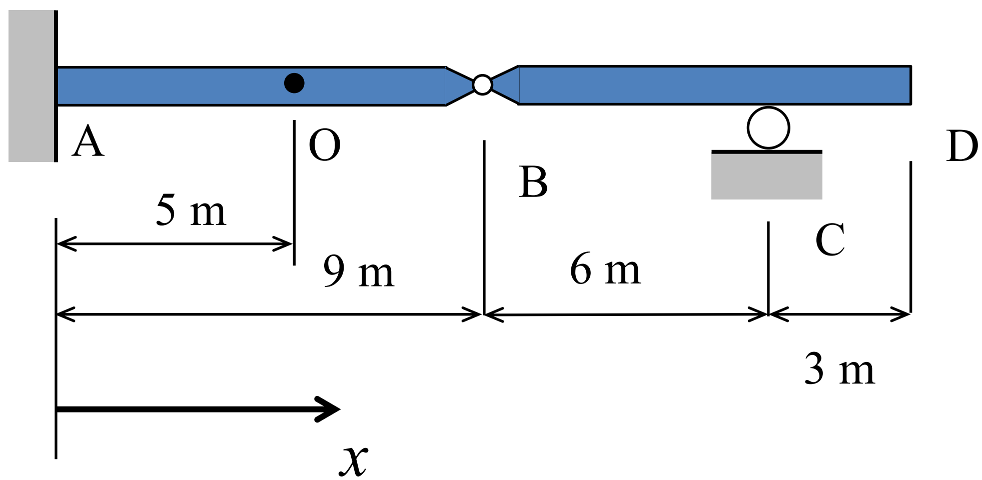


$x =$	V_O
0	0
5^- m	0
5^+ m	1
9^- m	1
12 m	0.5
15 m	0
18 m	– 0.5





Plot the Influence Line for M_O



$x =$	M_O
0	0
5^- m	0
5^+ m	0
9^- m	– 4 m
12 m	– 2 m
15 m	0
18 m	2 m