Using Beam Influence Lines Example

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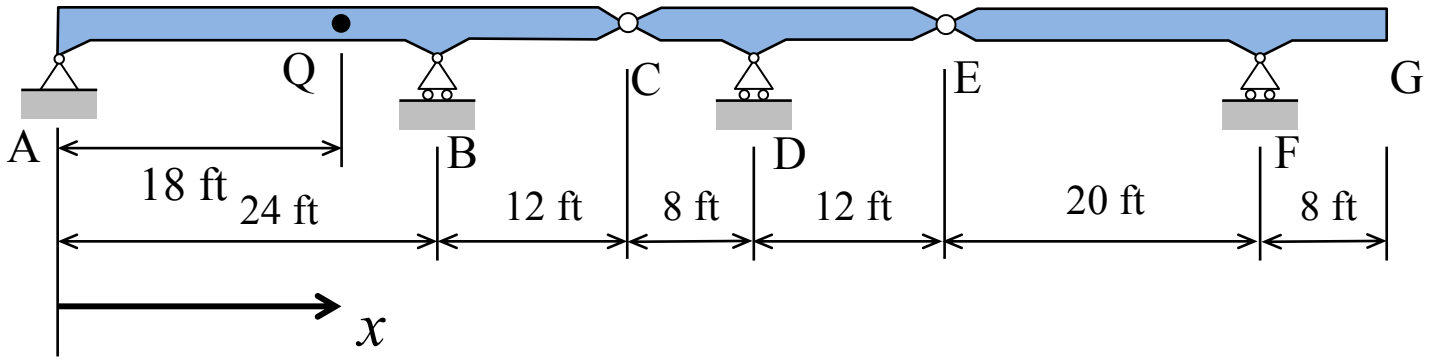
Example Problem

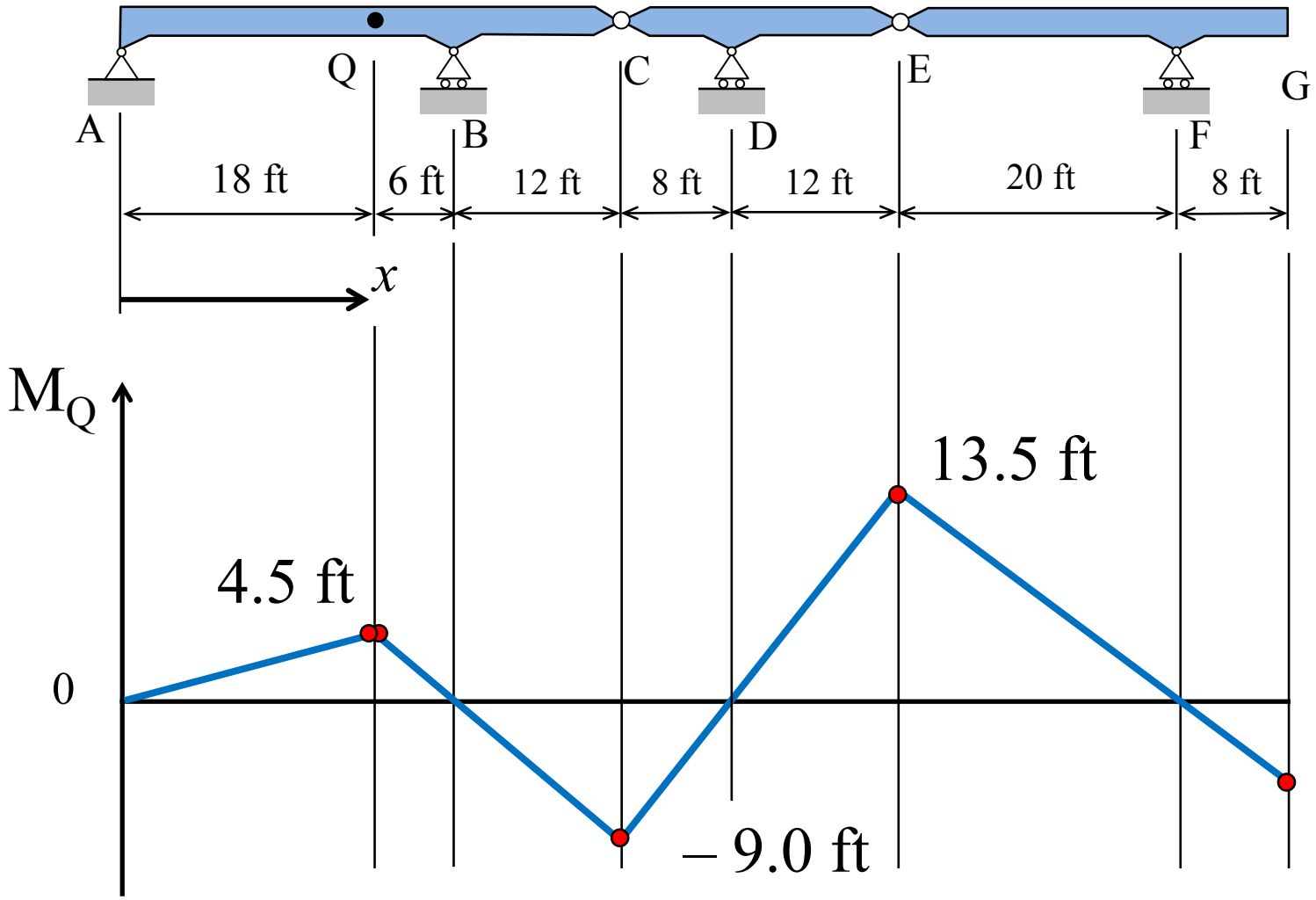


The hinged concrete beam from our previous example is subjected to a uniform dead load of 1.5 k/ft and a uniform live load of 5.5 k/ft or a point live load of 90 k. In order to design the steel reinforcement at point Q of the concrete beam, it is desired to find the following:

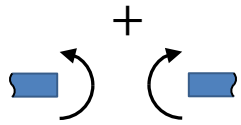
1. The maximum positive bending moment at point Q due to the to the uniform live load and dead load;
2. The maximum positive bending moment at point Q due to the to the point live load and dead load;
3. The maximum negative bending moment at point Q due to the uniform live live load and dead load.
4. The maximum negative bending moment at point Q due to the uniform point live load and dead load.

To place the live loads we need to construct the influence line for the bending moment at point Q. In a previous example, we constructed the M_Q influence line.

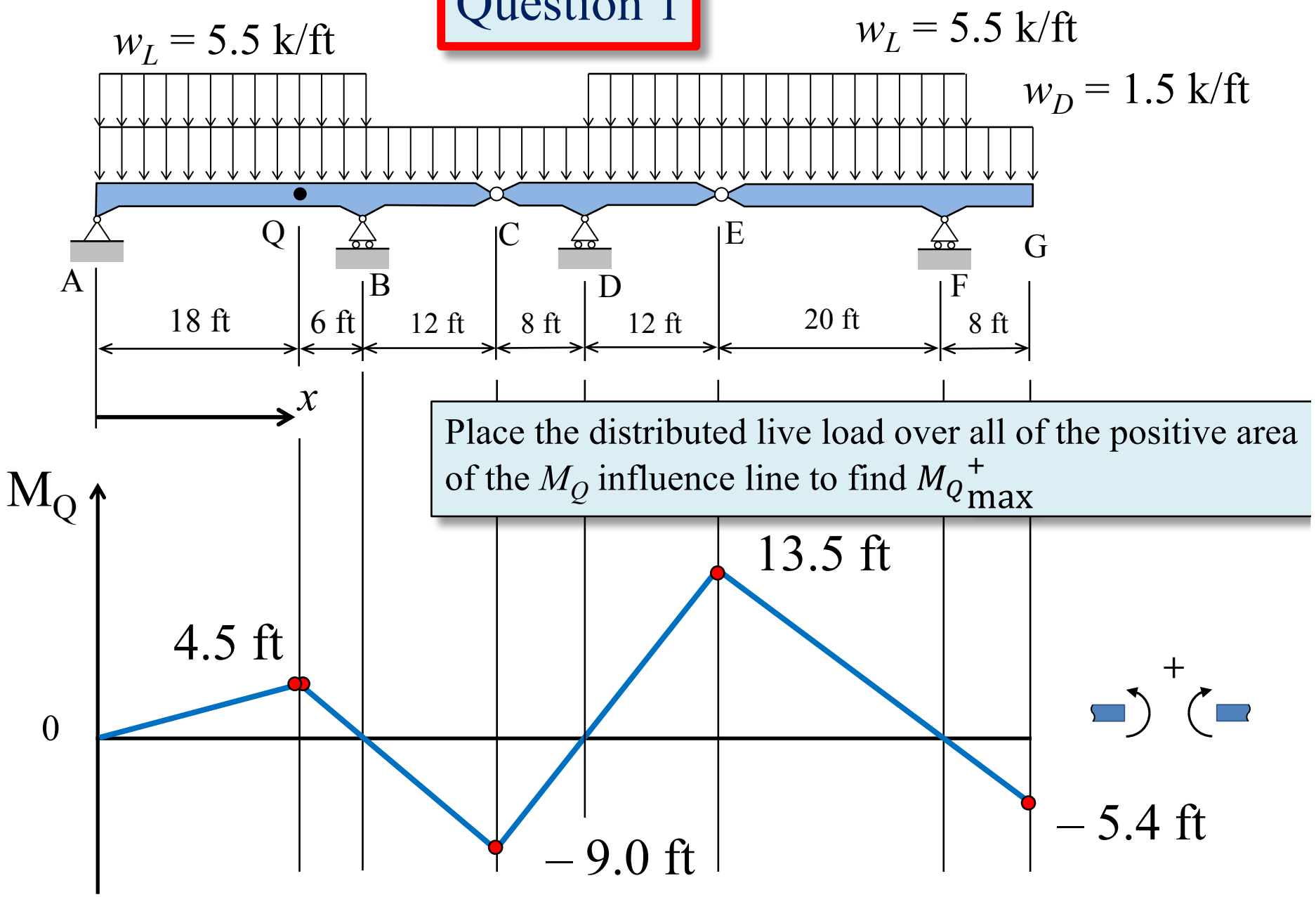
M_Q Influence Line



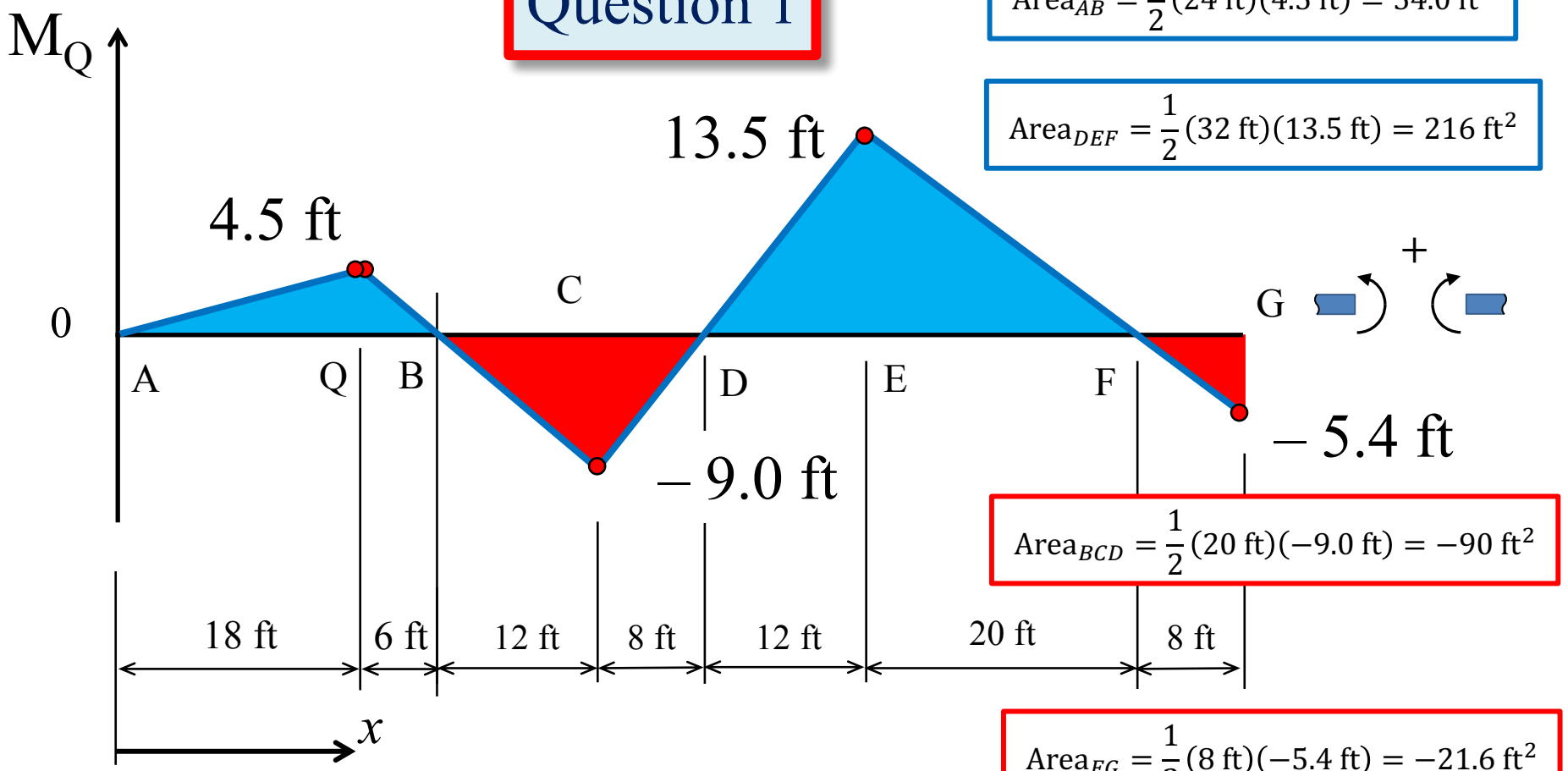
x =	M_Q
0	0
18^- ft	4.5 ft
18^+ ft	4.5 ft
24 ft	0
36 ft	– 9.0 ft
44 ft	0
56 ft	13.5 ft
76 ft	0
84 ft	– 5.4 ft



Question 1


w_L = 5.5 k/ft
w_L = 5.5 k/ft
w_D = 1.5 k/ft
A
Q
B
C
D
E
F
G
18 ft
6 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the distributed live load over all of the positive area of the M_Q influence line to find $M_{Q\,\max}^{+}$
M_Q
13.5 ft
4.5 ft
0
+
– 9.0 ft
– 5.4 ft

Question 1



$$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(4.5\text{ ft}) = 54.0\text{ ft}^2$$

$$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(13.5\text{ ft}) = 216\text{ ft}^2$$

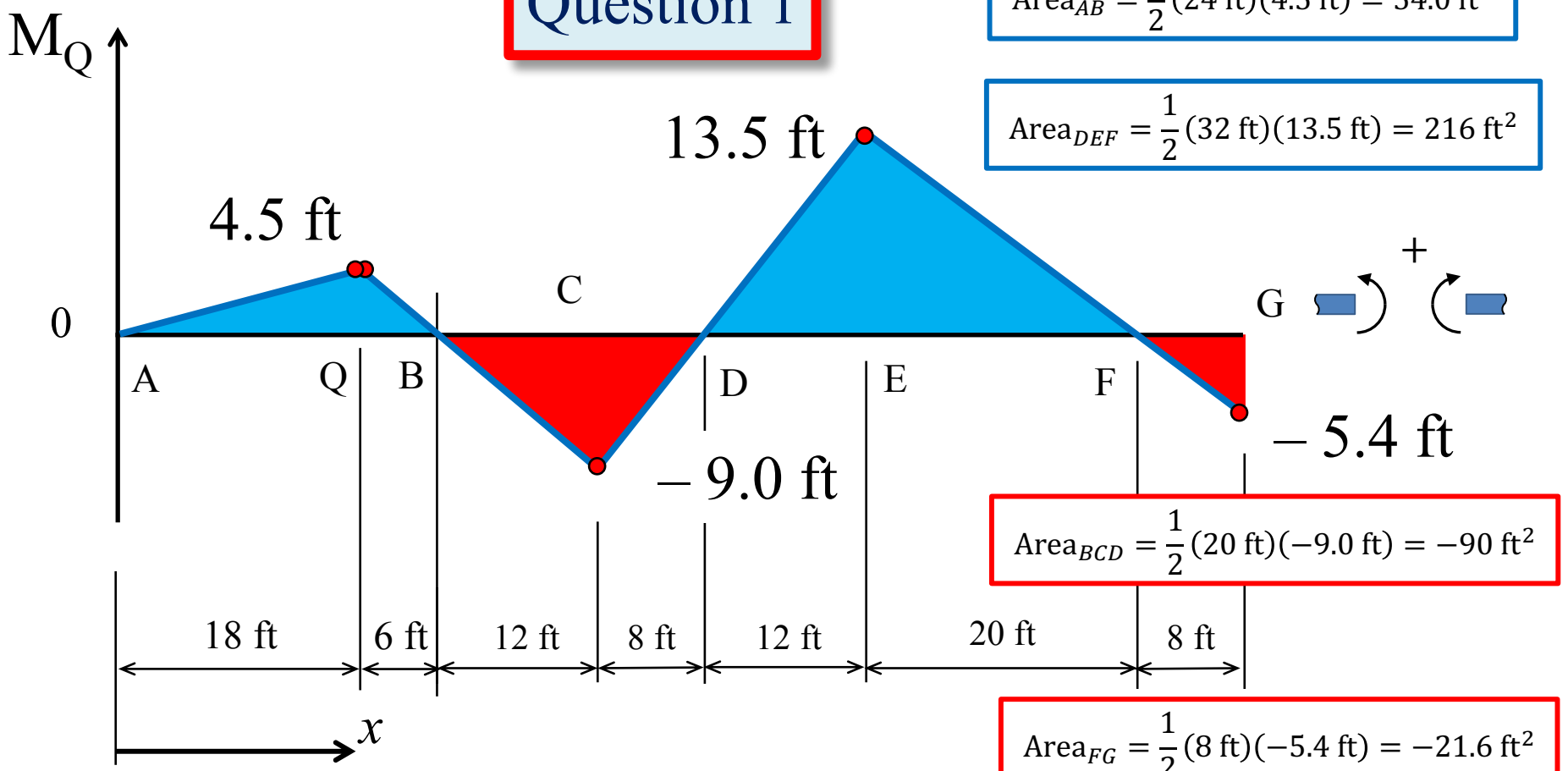
$$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-9.0\text{ ft}) = -90\text{ ft}^2$$

$$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-5.4\text{ ft}) = -21.6\text{ ft}^2$$

$$M_{Q_D} = (1.5\text{ k/ft})[54.0\text{ ft}^2 - 90\text{ ft}^2 + 216\text{ ft}^2 - 21.6\text{ ft}^2]$$

$$M_{Q_D} = (1.5\text{ k/ft})[158.4\text{ ft}^2] = 237.6\text{ k–ft}$$

Question 1



$$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(4.5\text{ ft}) = 54.0\text{ ft}^2$$

$$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(13.5\text{ ft}) = 216\text{ ft}^2$$

$$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-9.0\text{ ft}) = -90\text{ ft}^2$$

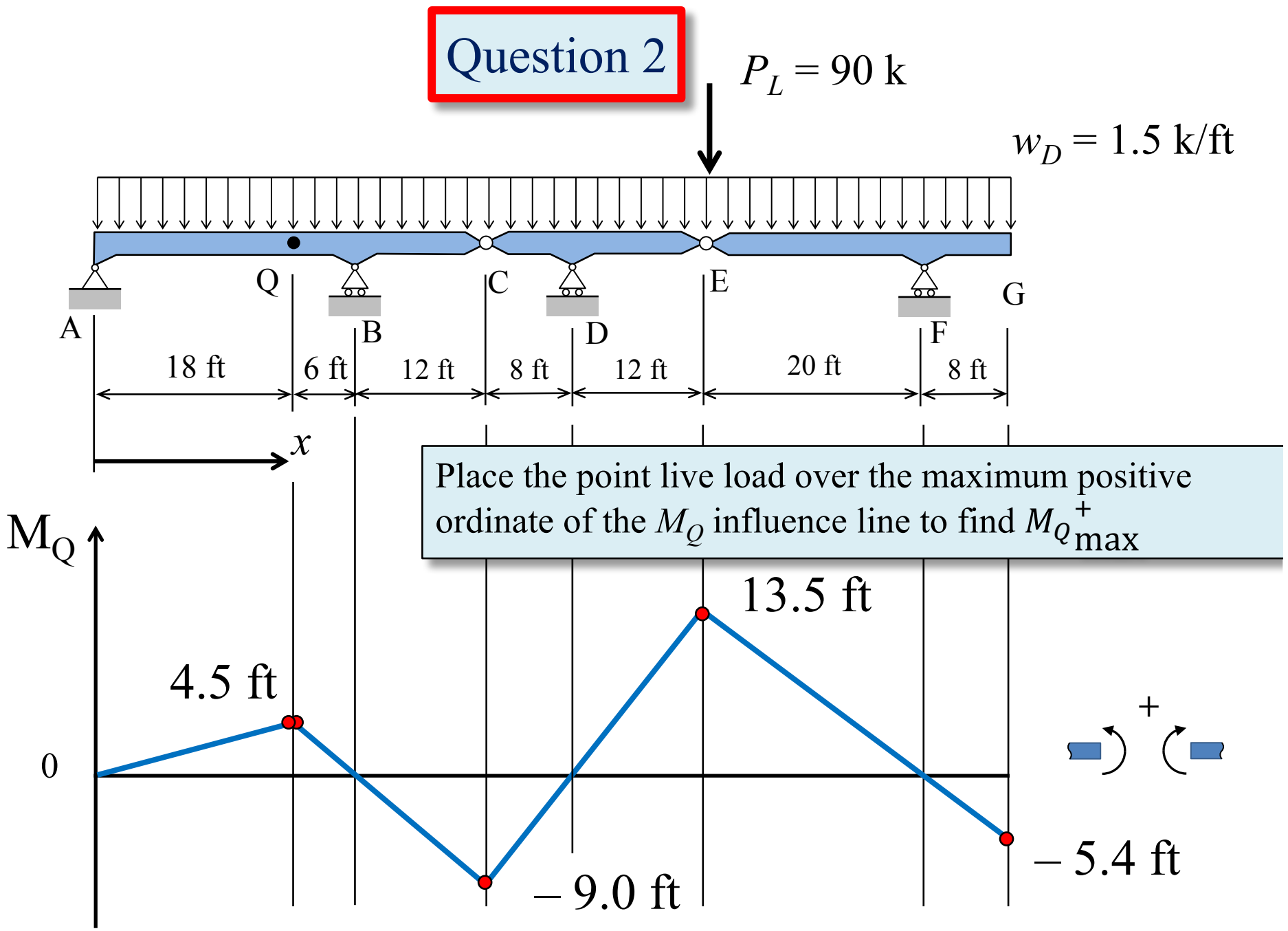
$$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-5.4\text{ ft}) = -21.6\text{ ft}^2$$

$$M^+_{Qmax_L} = (5.5\text{ k/ft})[54.0\text{ ft}^2 + 216\text{ ft}^2] = 1485.0\text{ k–ft}$$

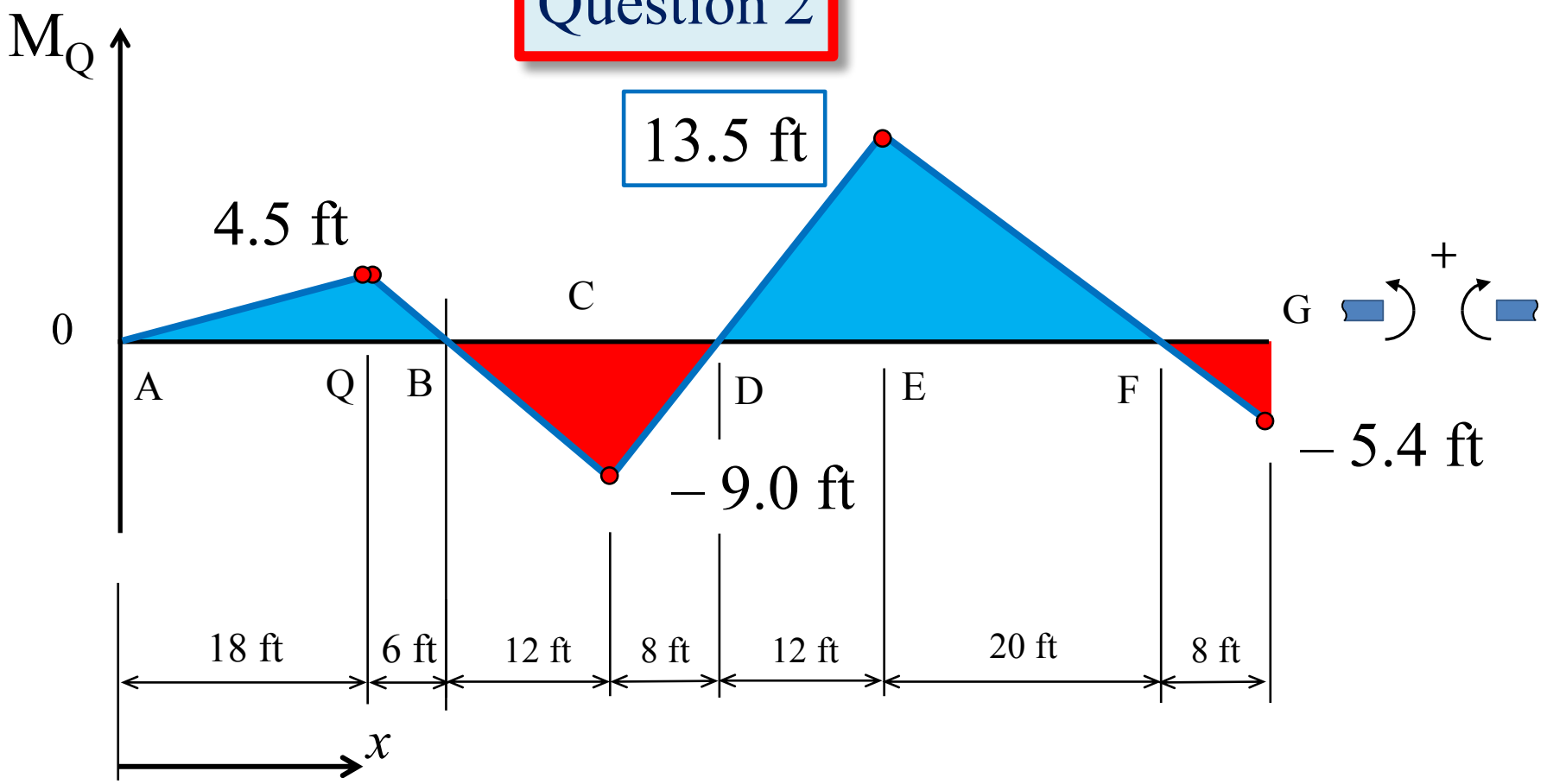
$$M_{Q_D} = 237.6\text{ k–ft}$$

$$M^+_{Qmax} = 237.6\text{ k–ft} + 1485.0\text{ k–ft} = 1722.6\text{ k–ft}$$

Question 2


$P_L = 90$ k
$w_D = 1.5$ k/ft
A
Q
B
C
D
E
F
G
18 ft
6 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the point live load over the maximum positive ordinate of the M_Q influence line to find $M_{Q\,\max}^{+}$
M_Q
13.5 ft
4.5 ft
0
+
– 5.4 ft
– 9.0 ft

Question 2



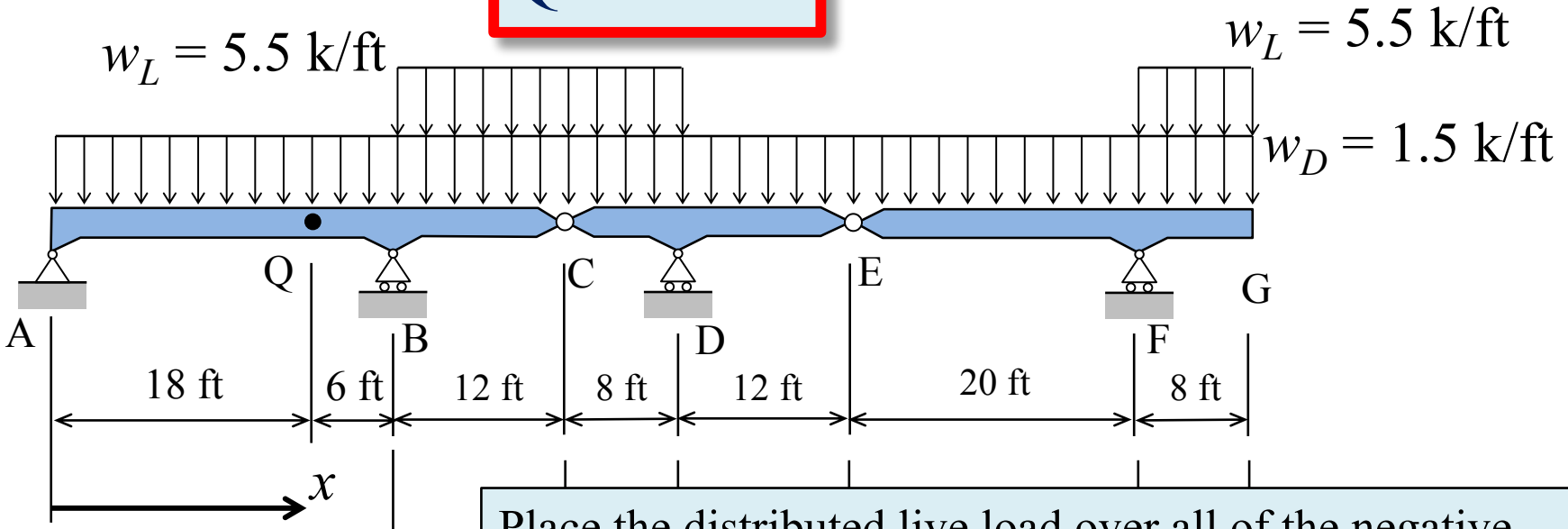
$M^{+}_{Qmax_L} = (90\text{ k})[13.5\text{ ft}] = 1215\text{ k–ft}$

$M_{Q_D} = 237.6\text{ k–ft}$

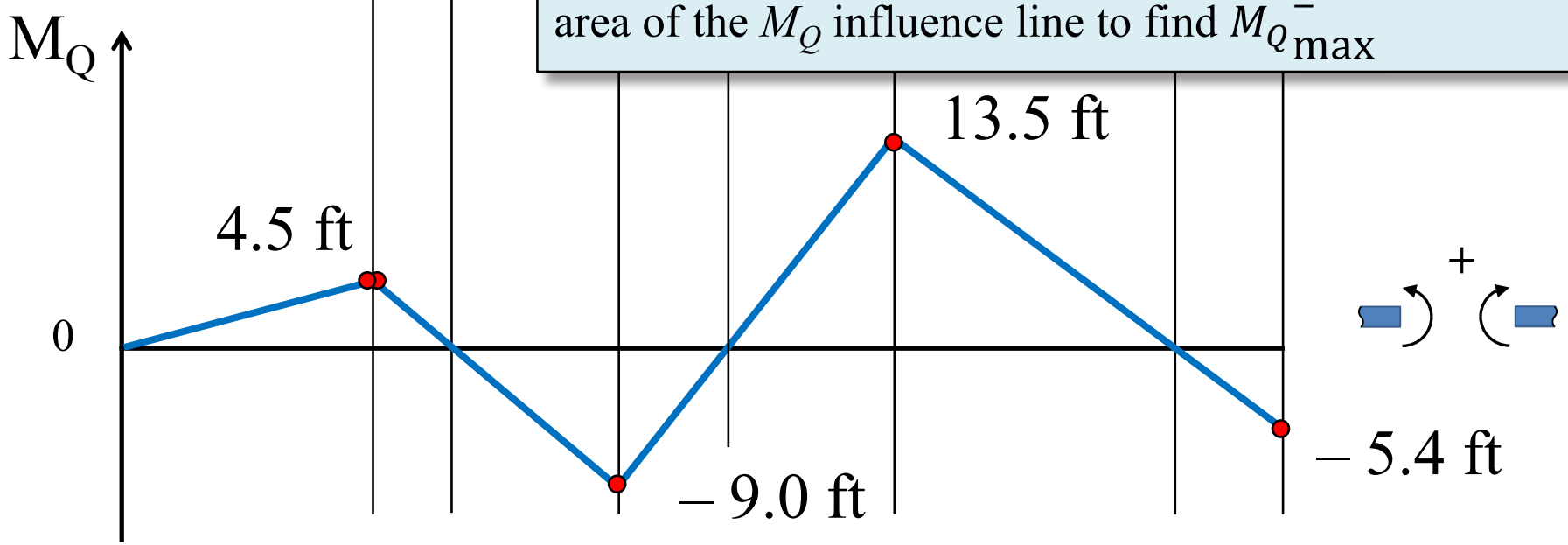
Dead load is fixed, so M_{QD} remains the same.

$M^{+}_{Qmax} = 237.6\text{ k–ft} + 1215\text{ k–ft} = 1452.6\text{ k–ft}$

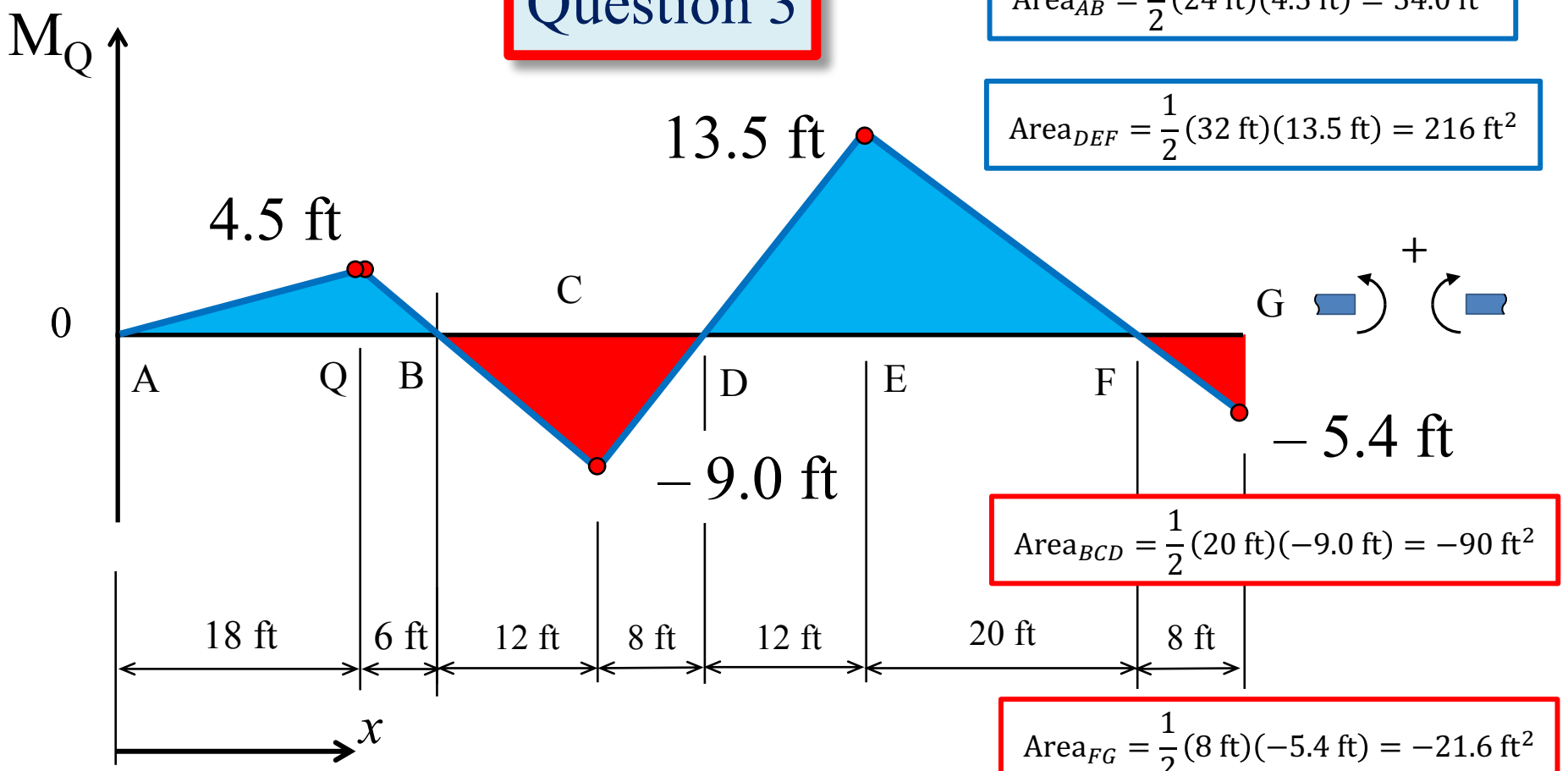
Question 3



Place the distributed live load over all of the negative area of the M_Q influence line to find $M_{Q_{max}}^{-}$



Question 3



$$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(4.5\text{ ft}) = 54.0\text{ ft}^2$$

$$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(13.5\text{ ft}) = 216\text{ ft}^2$$

$$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-9.0\text{ ft}) = -90\text{ ft}^2$$

$$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-5.4\text{ ft}) = -21.6\text{ ft}^2$$

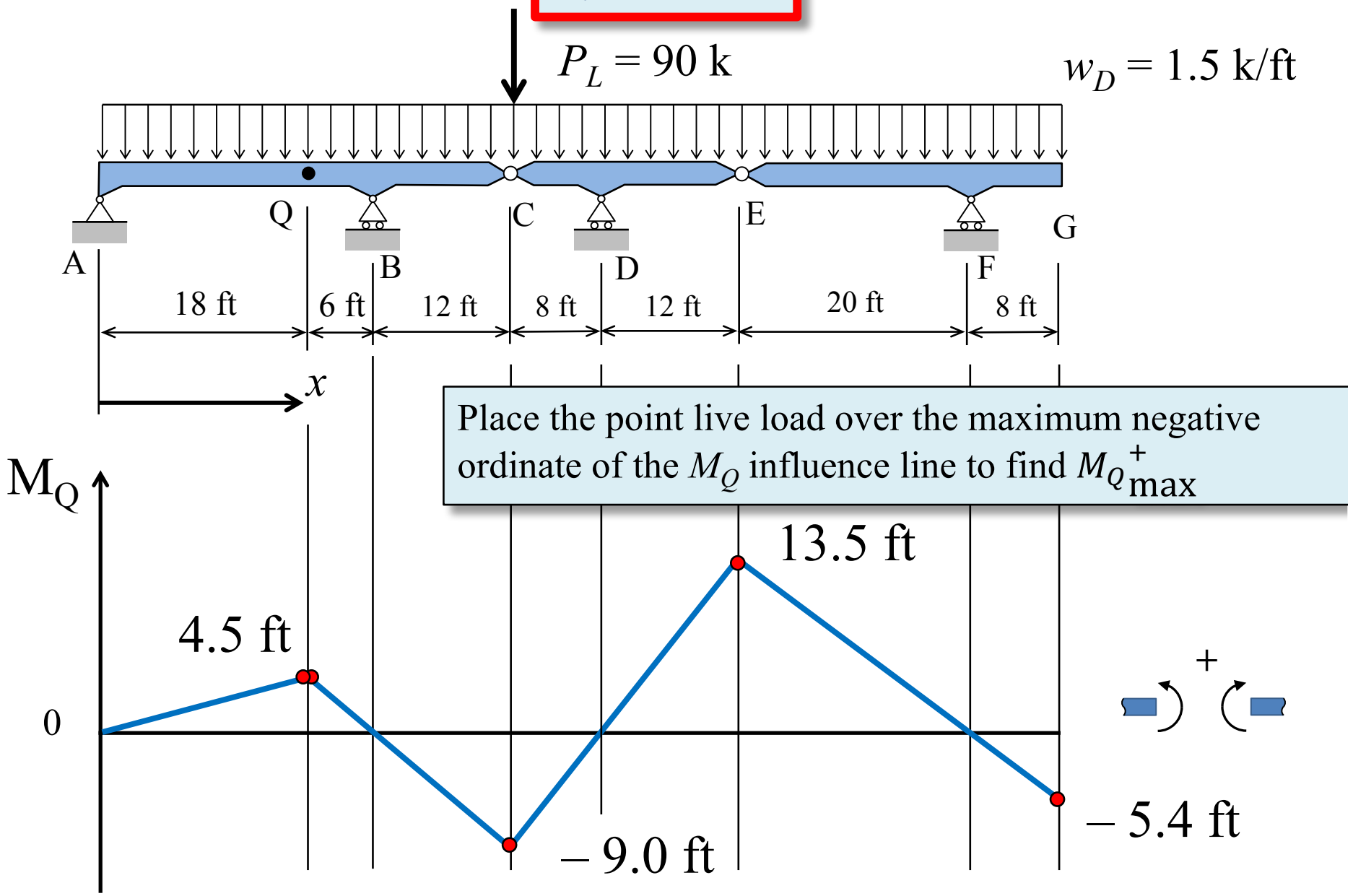
$$M^-_{Qmax_L} = (5.5\text{ k/ft})[-90\text{ ft}^2 - 21.6\text{ ft}^2] = -613.8\text{ k–ft}$$

$$M_{Q_D} = 237.6\text{ k–ft}$$

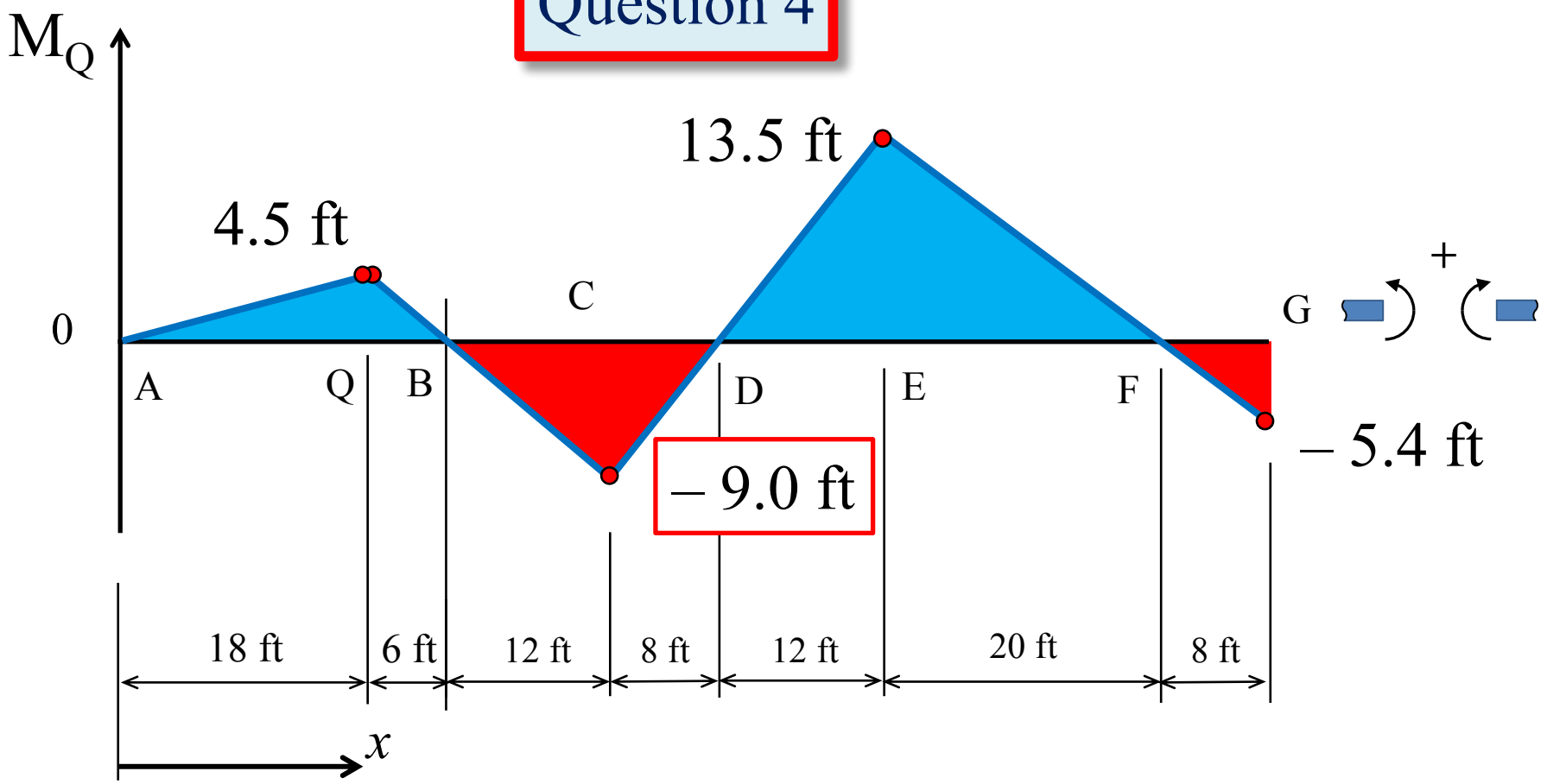
Dead load is fixed, so M_{QD} remains the same.

$$M^-_{Qmax} = 237.6\text{ k–ft} - 613.8\text{ k–ft} = -376.2\text{ k–ft}$$

Question 4


$P_L = 90$ k
$w_D = 1.5$ k/ft
A
Q
B
C
D
E
F
G
18 ft
6 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the point live load over the maximum negative ordinate of the M_Q influence line to find $M_{Q\,\max}^{+}$
M_Q
13.5 ft
4.5 ft
0
+
– 5.4 ft
– 9.0 ft

Question 4



$$M^{+}_{Qmax_L} = (90\text{ k})[-9.0\text{ ft}] = -810\text{ k–ft}$$

$$M_{Q_D} = 237.6\text{ k–ft}$$

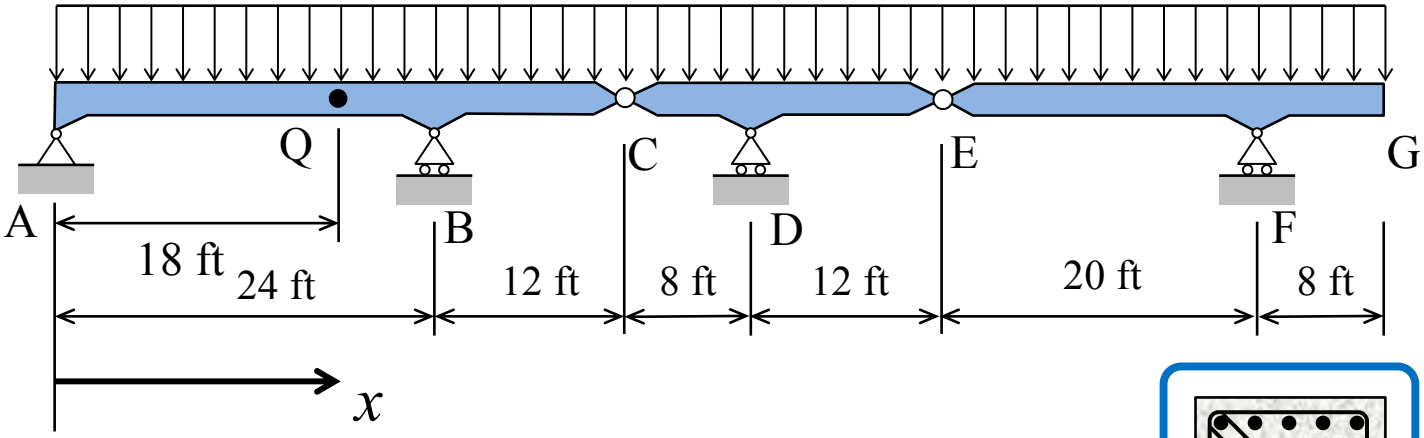
Dead load is fixed, so M_{QD} remains the same.

$$M^{+}_{Qmax} = 237.6\text{ k–ft} - 810\text{ k–ft} = -572.4\text{ k–ft}$$

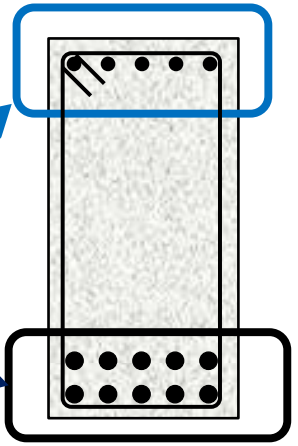
Summary of Results for M_{Qmax}

$P_L = 90$ k **OR** $w_L = 5.5$ k/ft

$w_D = 1.5$ k/ft

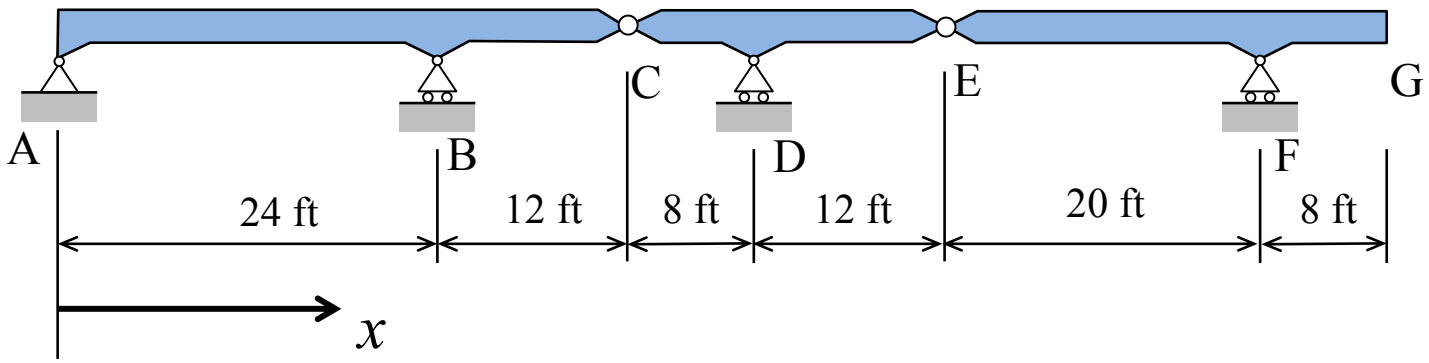


L	M_{Qmax}
5.5 k/ft	**1722.6 k-ft**
90 k	1452.6 k-ft
5.5 k/ft	–376.2 k-ft
90 k	**–572.4 k-ft**



Concrete Beam Section at Point Q

Another Example

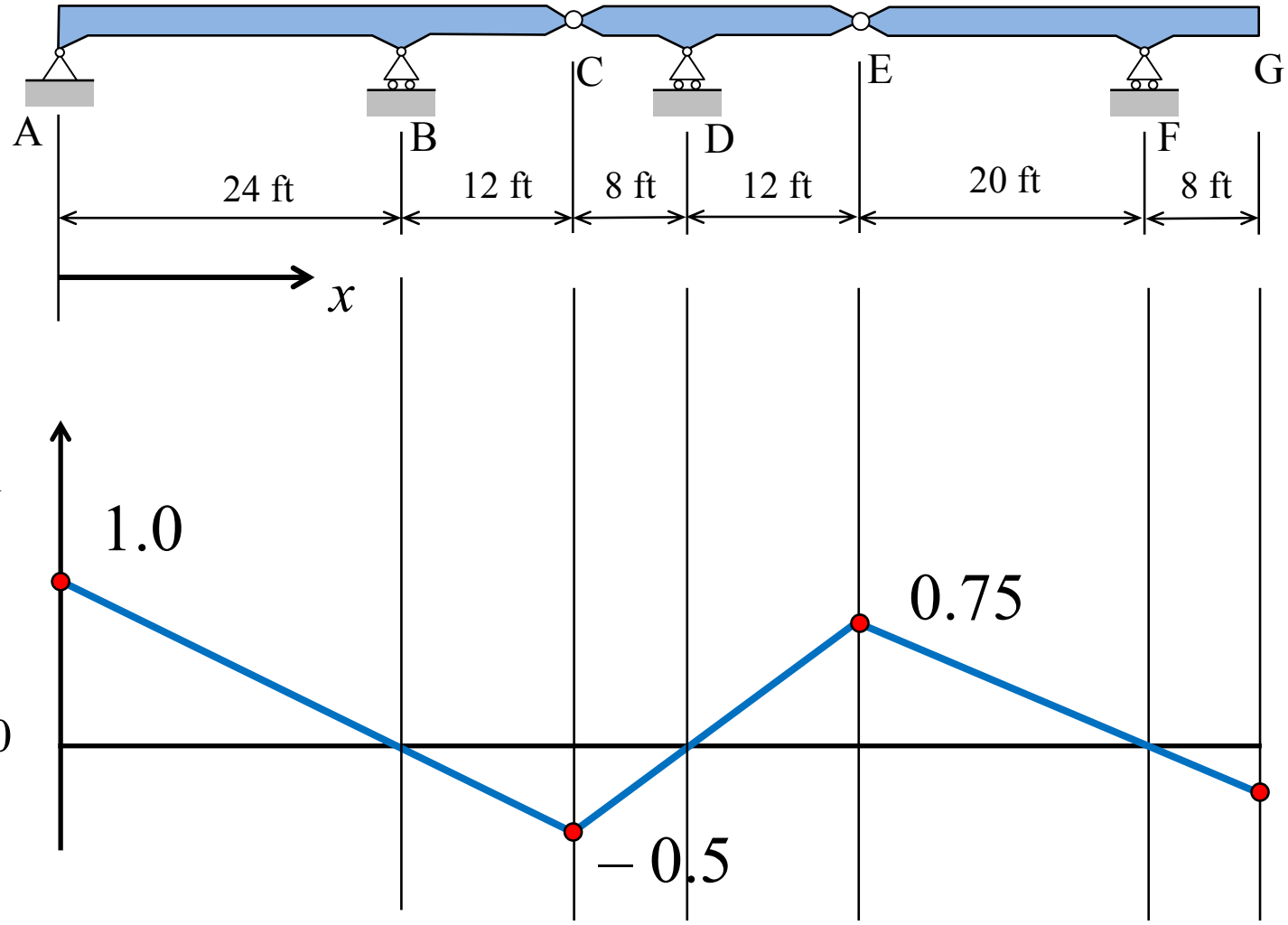


The hinged concrete beam from our previous example is subjected to a uniform dead load of 1.5 k/ft and a uniform live load of 5.5 k/ft or a point live load of 90 k. In order to design the pin support at point A, it is desired to find the following:

1. The maximum positive vertical reaction at A due to the to the uniform live load and dead load;
2. The maximum positive vertical reaction at A due to the point live load and dead load;
3. The maximum negative vertical reaction at A due to the uniform live live load and dead load.
4. The maximum negative vertical reaction at A due to the point live load and dead load.

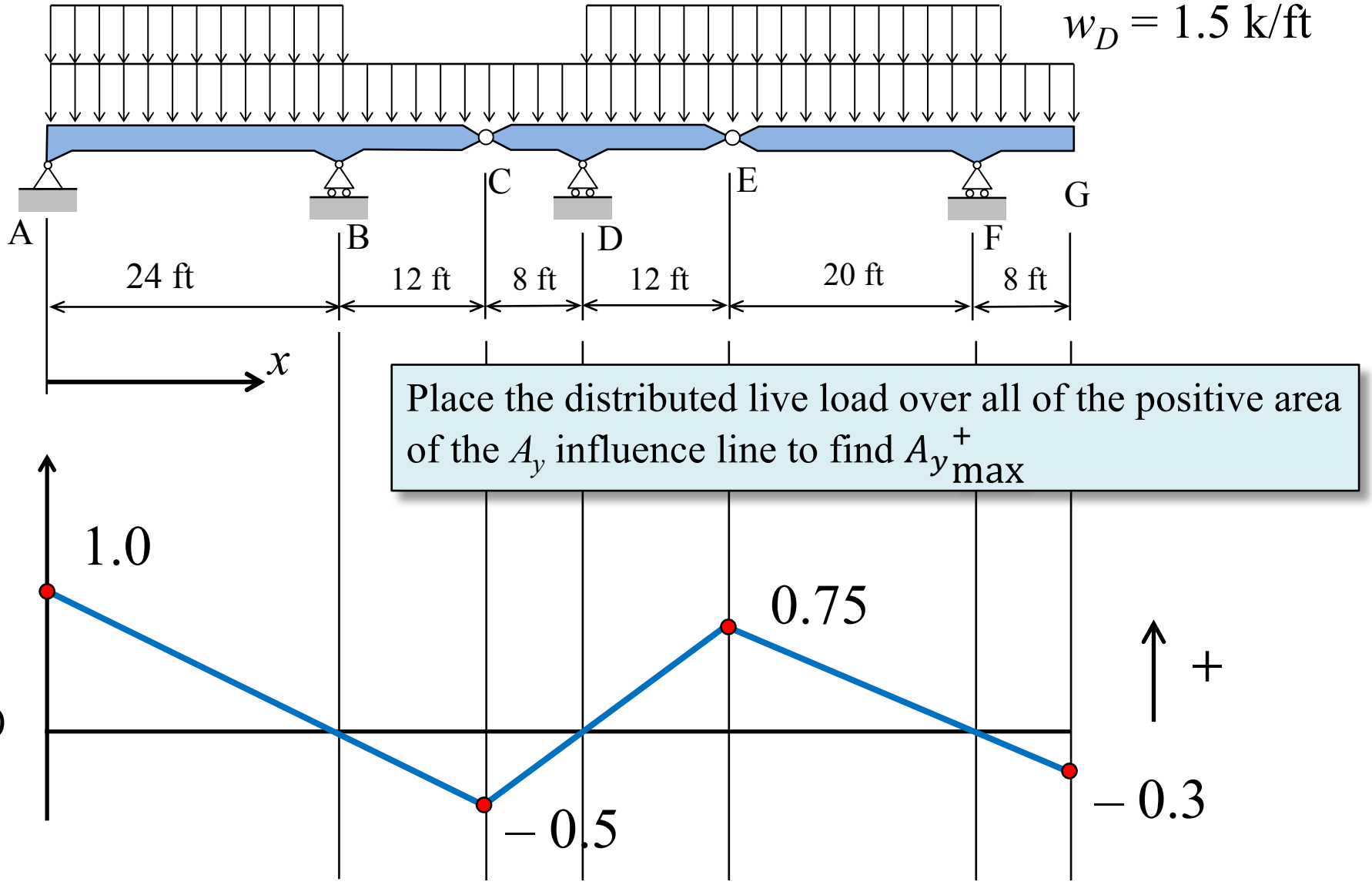
To place the live loads we need to construct the influence line for the vertical reaction at point A. In a previous example, we constructed the A_y influence line.

A_y Influence Line

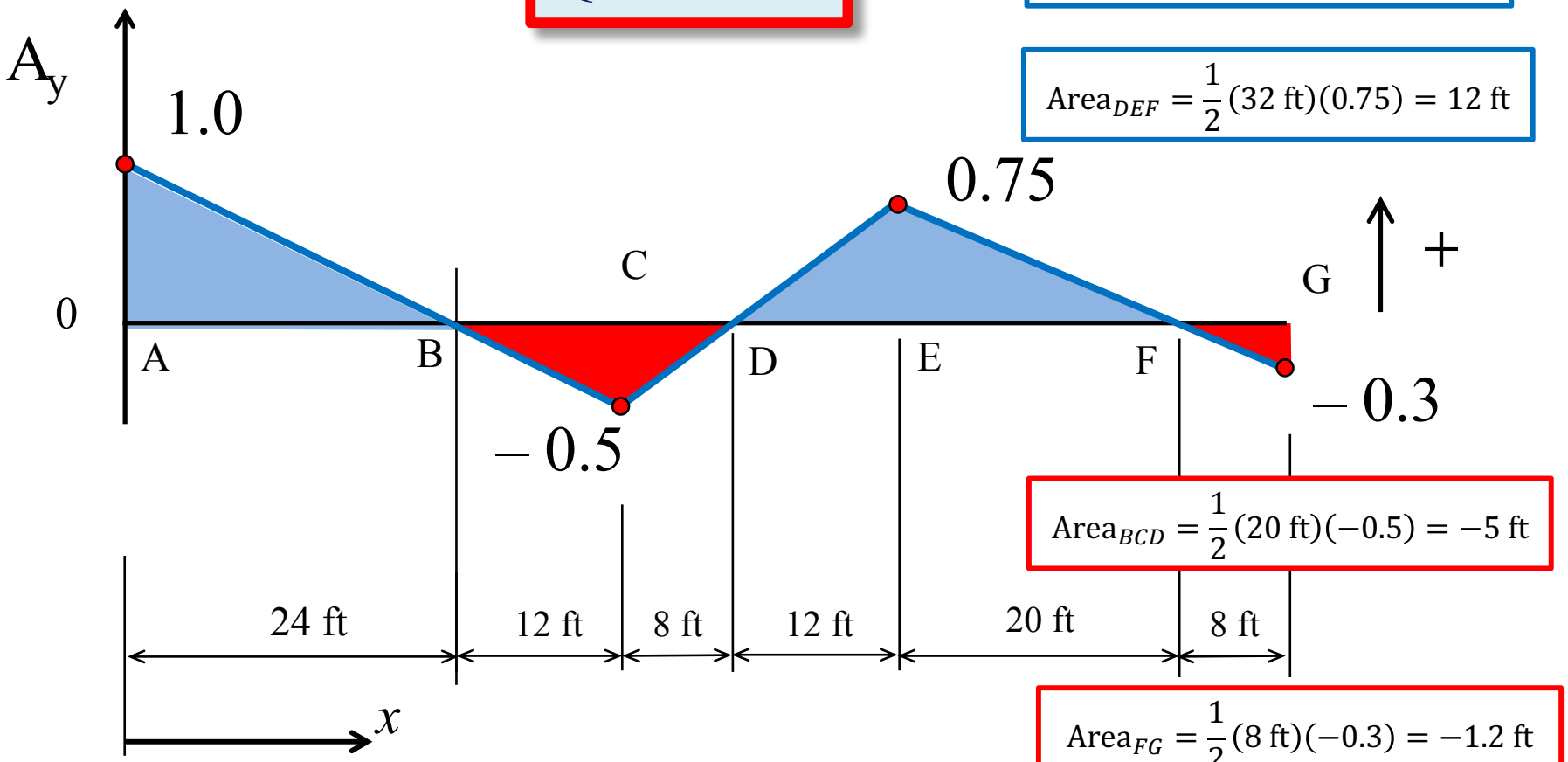


x =	A_y
0	1.0
24 ft	0
36 ft	– 0.5
44 ft	0
56 ft	0.75
76 ft	0
84 ft	– 0.3

Question 1


w_L = 5.5 k/ft
w_L = 5.5 k/ft
w_D = 1.5 k/ft
A
B
C
D
E
F
G
24 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the distributed live load over all of the positive area of the A_y influence line to find $A_{y_{max}}^{+}$
A_y
1.0
0
0.75
– 0.5
– 0.3
+

Question 1



$$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(1.0) = 12\text{ ft}$$

$$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(0.75) = 12\text{ ft}$$

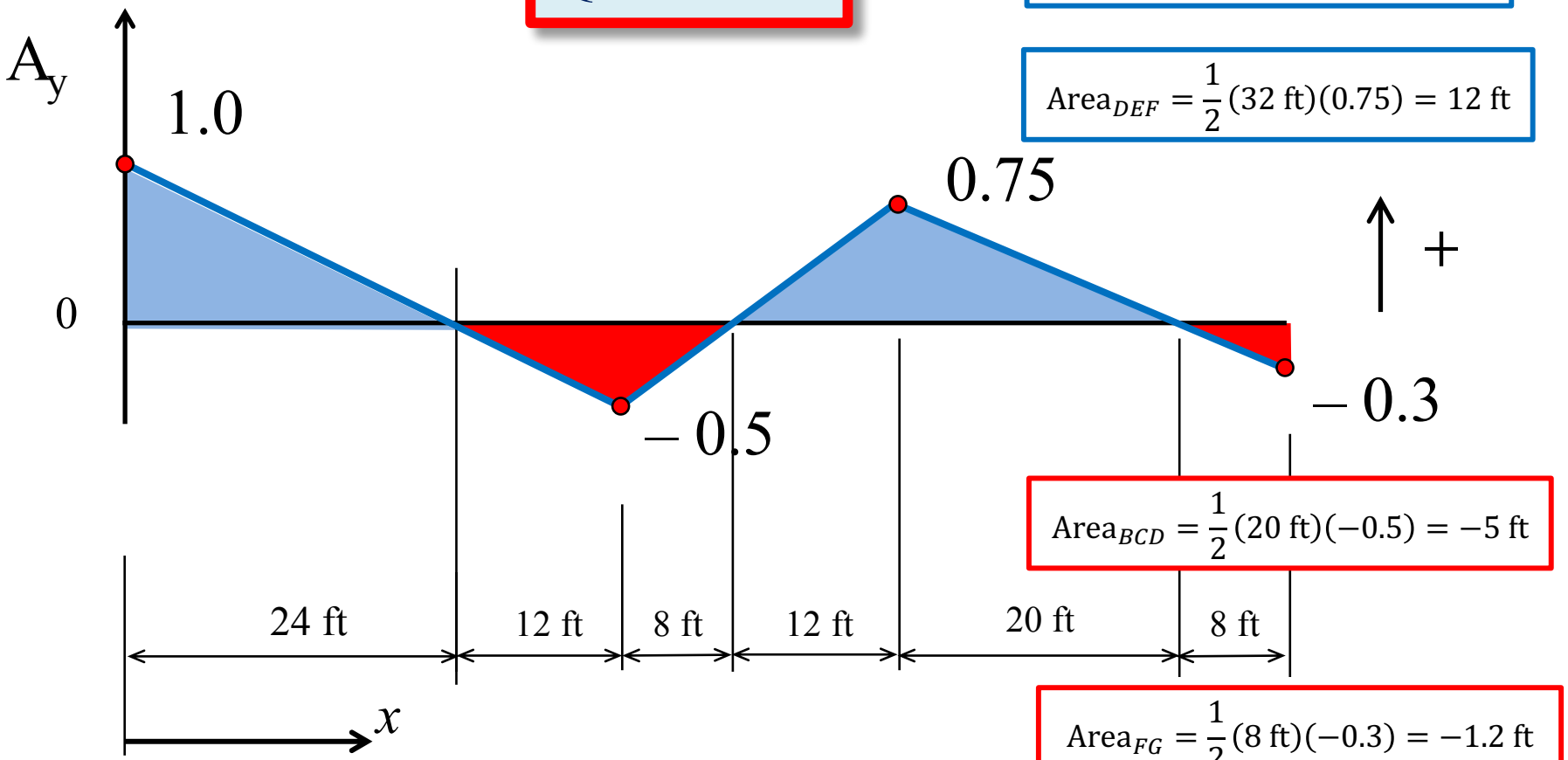
$$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-0.5) = -5\text{ ft}$$

$$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-0.3) = -1.2\text{ ft}$$

$$A_{y_D} = (1.5\text{ k/ft})[12\text{ ft} - 5\text{ ft} + 12\text{ ft} - 1.2\text{ ft}]$$

$$A_{y_D} = (1.5\text{ k/ft})[17.8\text{ ft}] = 26.7\text{ k}$$

Question 1



$$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(1.0) = 12\text{ ft}$$

$$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(0.75) = 12\text{ ft}$$

$$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-0.5) = -5\text{ ft}$$

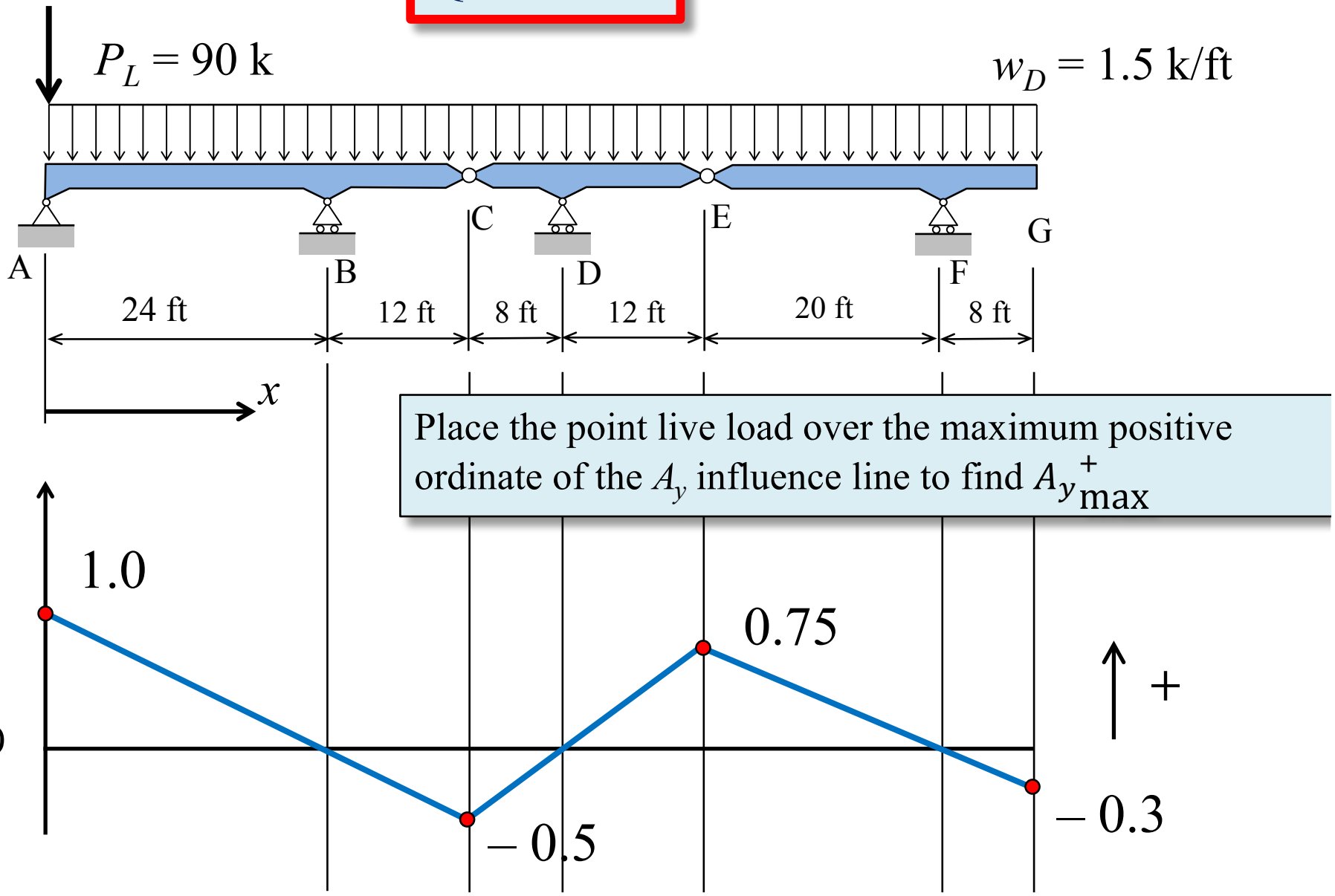
$$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-0.3) = -1.2\text{ ft}$$

$$A^{+}_{ymax_L} = (5.5\text{ k/ft})[12\text{ ft} + 12\text{ ft}] = 132\text{ k}$$

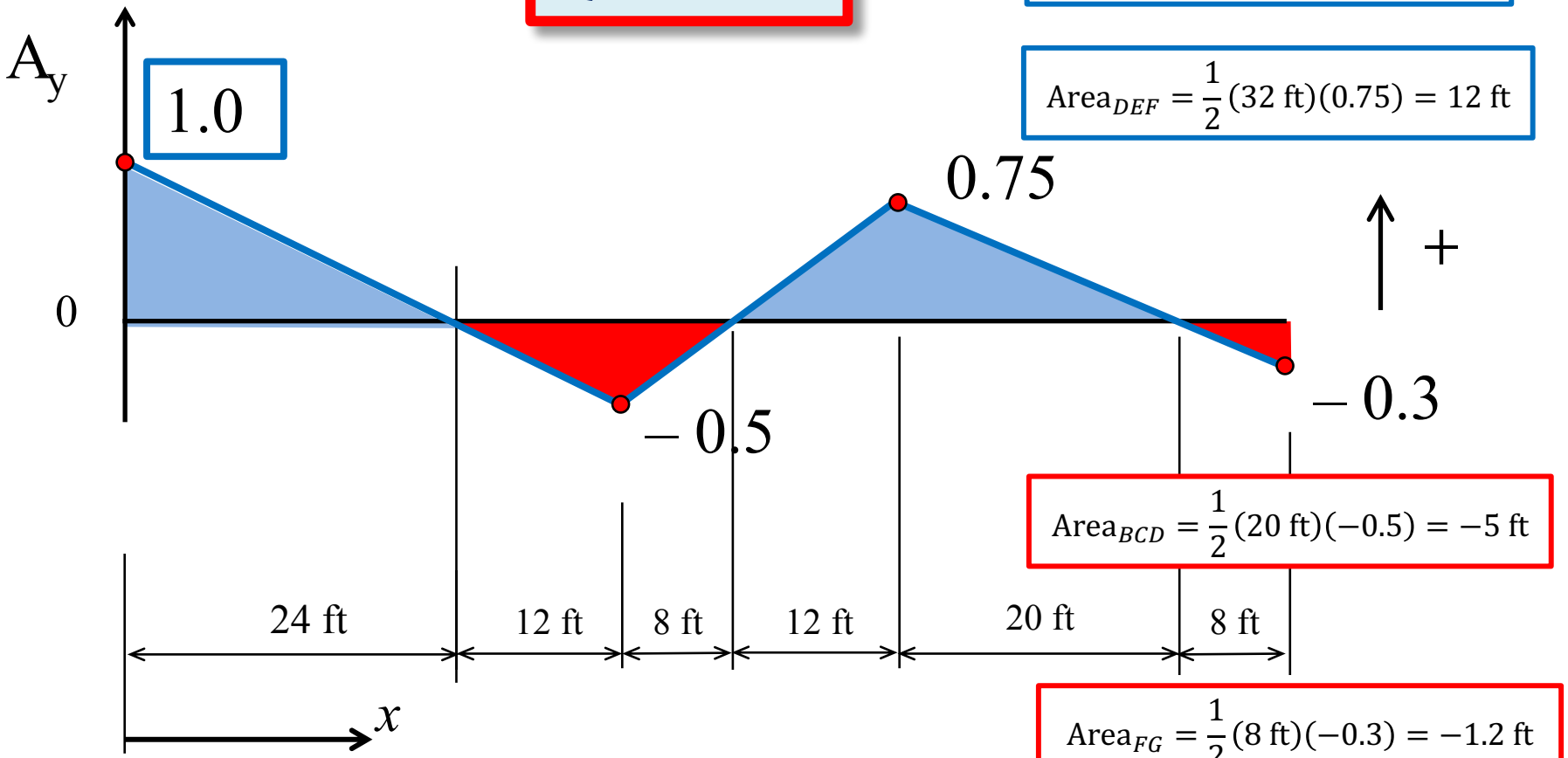
$$A_{y_D} = 26.7\text{ k}$$

$$A^{+}_{ymax} = 26.7\text{ k} + 132\text{ k} = 158.7\text{ k}$$

Question 2


$P_L = 90$ k
$w_D = 1.5$ k/ft
A
B
C
D
E
F
G
24 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the point live load over the maximum positive ordinate of the A_y influence line to find $A_{y\,\max}^{+}$
A_y
1.0
0.75
0
+
– 0.5
– 0.3

Question 2



$$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(1.0) = 12\text{ ft}$$

$$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(0.75) = 12\text{ ft}$$

$$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-0.5) = -5\text{ ft}$$

$$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-0.3) = -1.2\text{ ft}$$

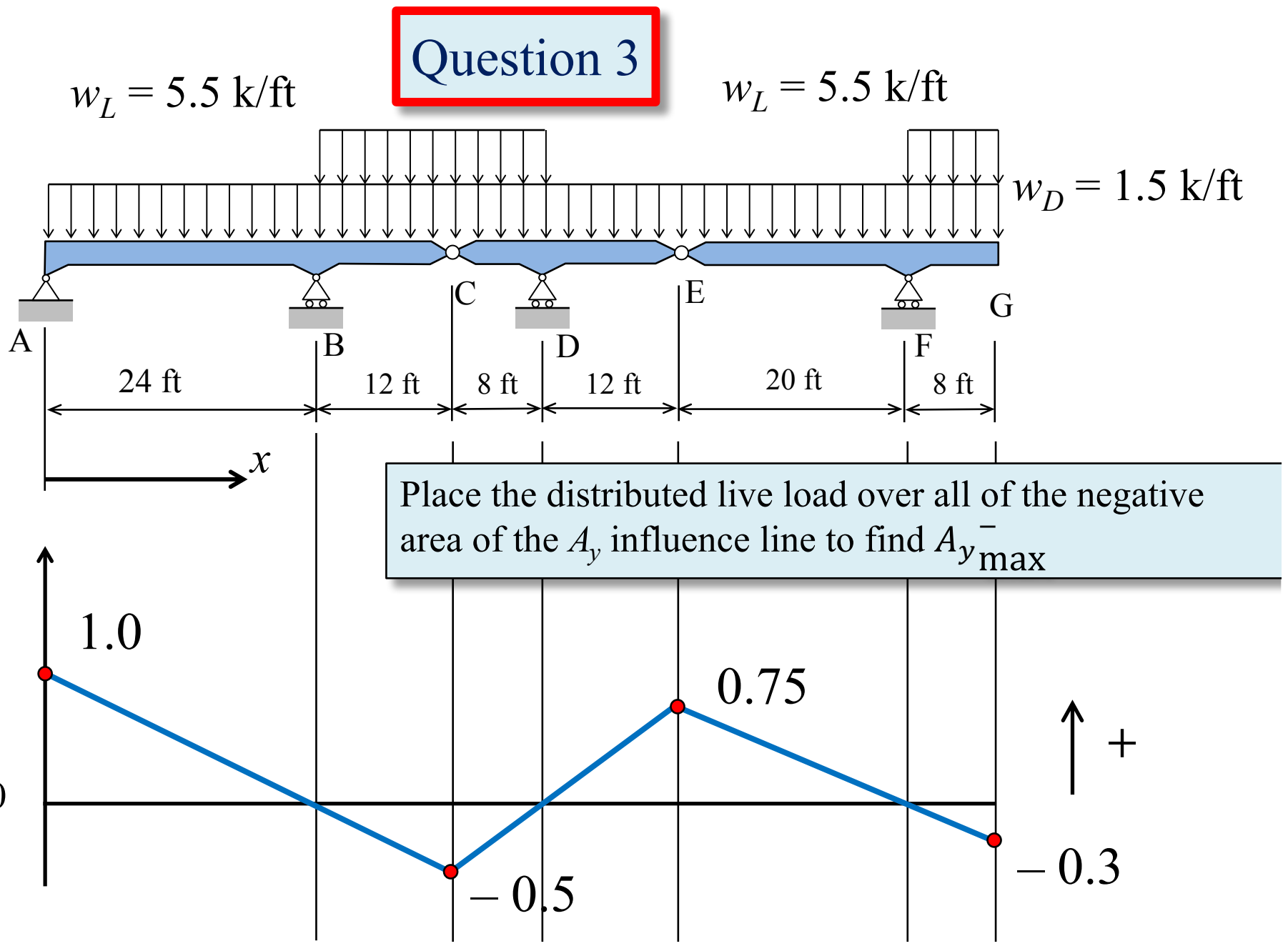
$$A^{+}_{ymax_L} = (90\text{ k})[1] = 90\text{ k}$$

$$A_{y_D} = 26.7\text{ k}$$

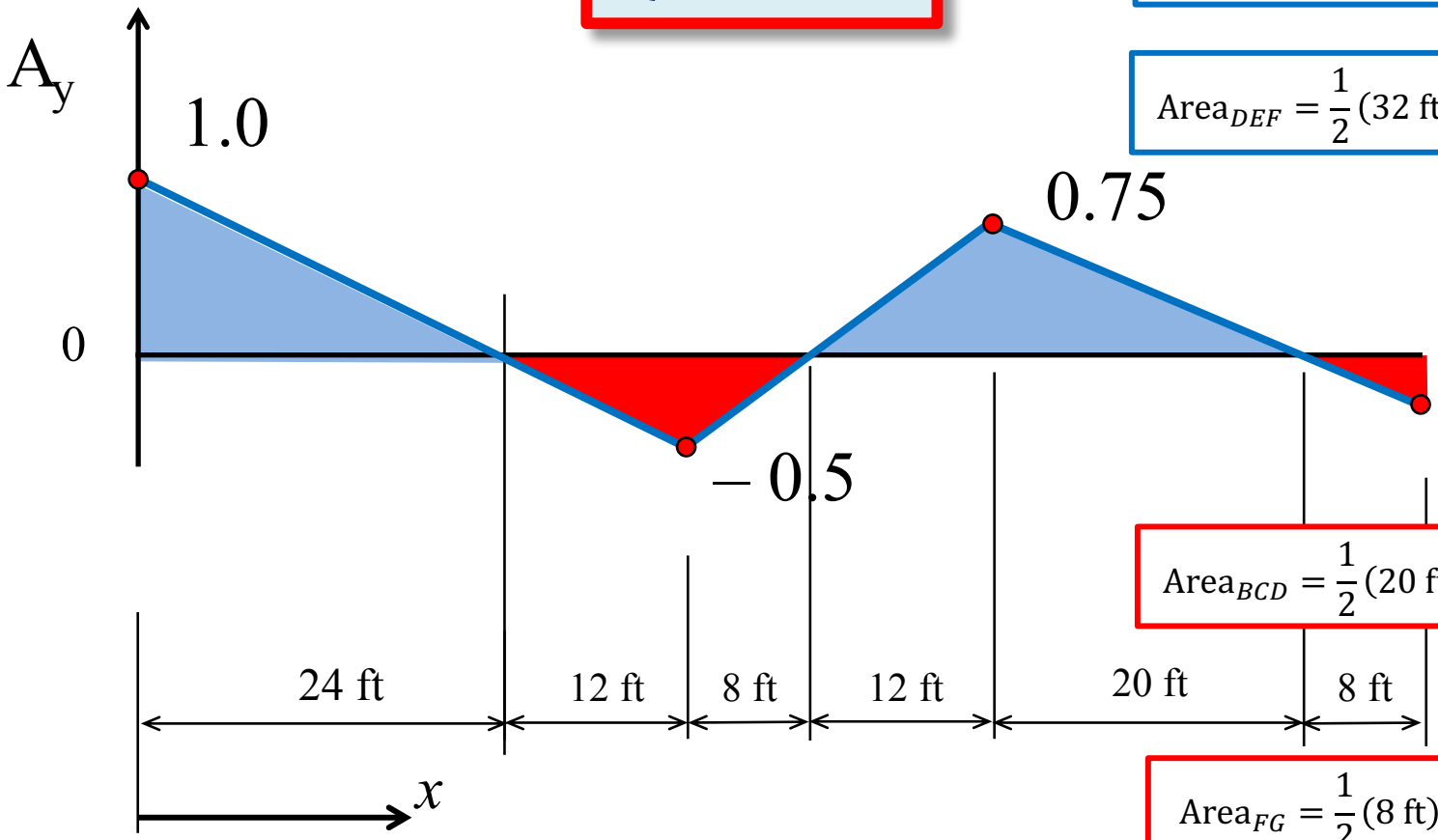
Dead load is fixed, so A_{yD} remains the same.

$$A^{+}_{ymax} = 26.7\text{ k} + 90\text{ k} = 116.7\text{ k}$$

Question 3


w_L = 5.5 k/ft
w_L = 5.5 k/ft
w_D = 1.5 k/ft
A
B
C
D
E
F
G
24 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the distributed live load over all of the negative area of the A_y influence line to find $A_{y_{max}}^{-}$
A_y
1.0
0.75
0
+
– 0.5
– 0.3

Question 3



$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(1.0) = 12\text{ ft}$

$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(0.75) = 12\text{ ft}$

$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-0.5) = -5\text{ ft}$

$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-0.3) = -1.2\text{ ft}$

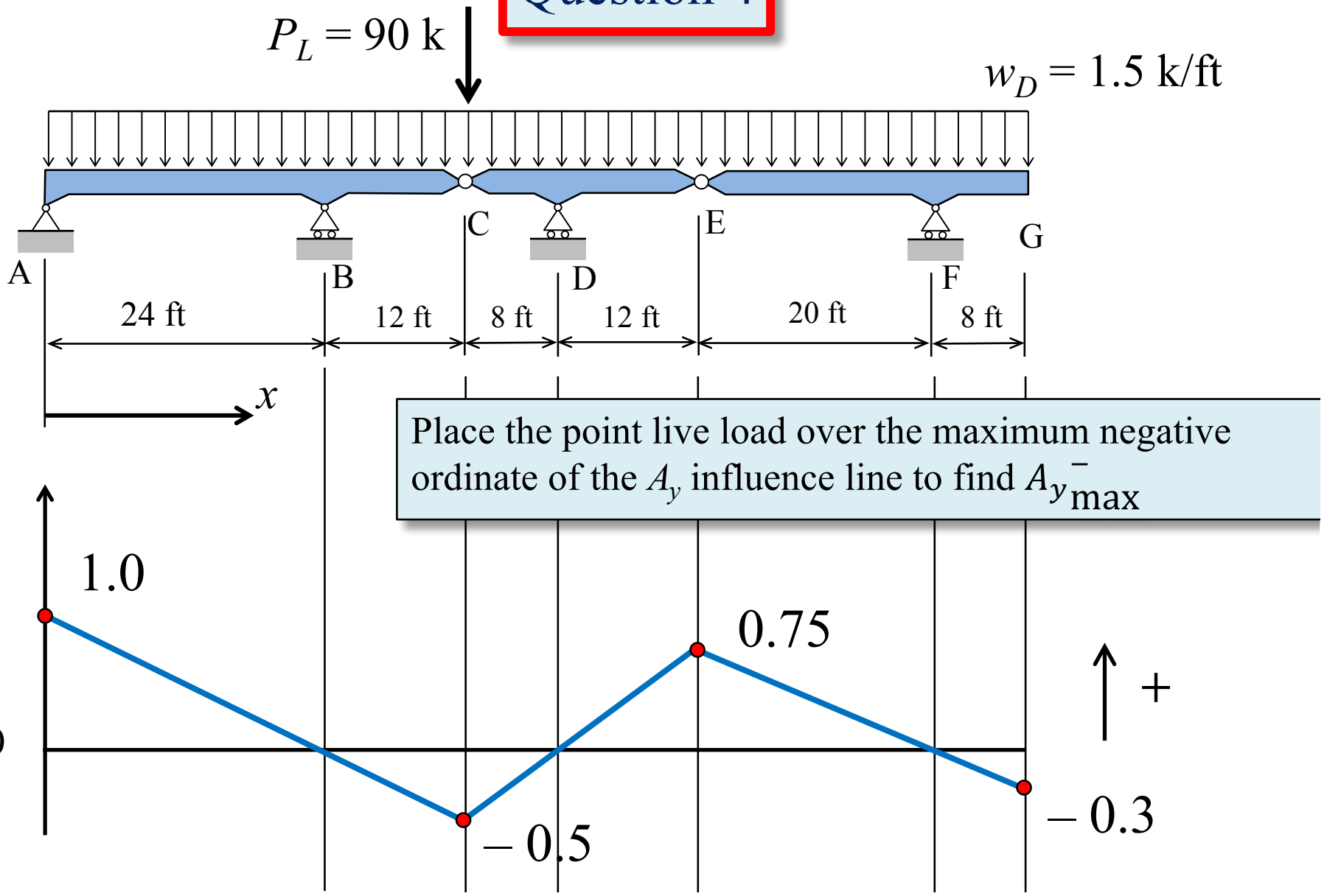
$A^{-}_{ymax_L} = (5.5\text{ k/ft})[-5\text{ ft} - 1.2\text{ ft}] = -34.1\text{ k}$

$A_{y_D} = 26.7\text{ k}$

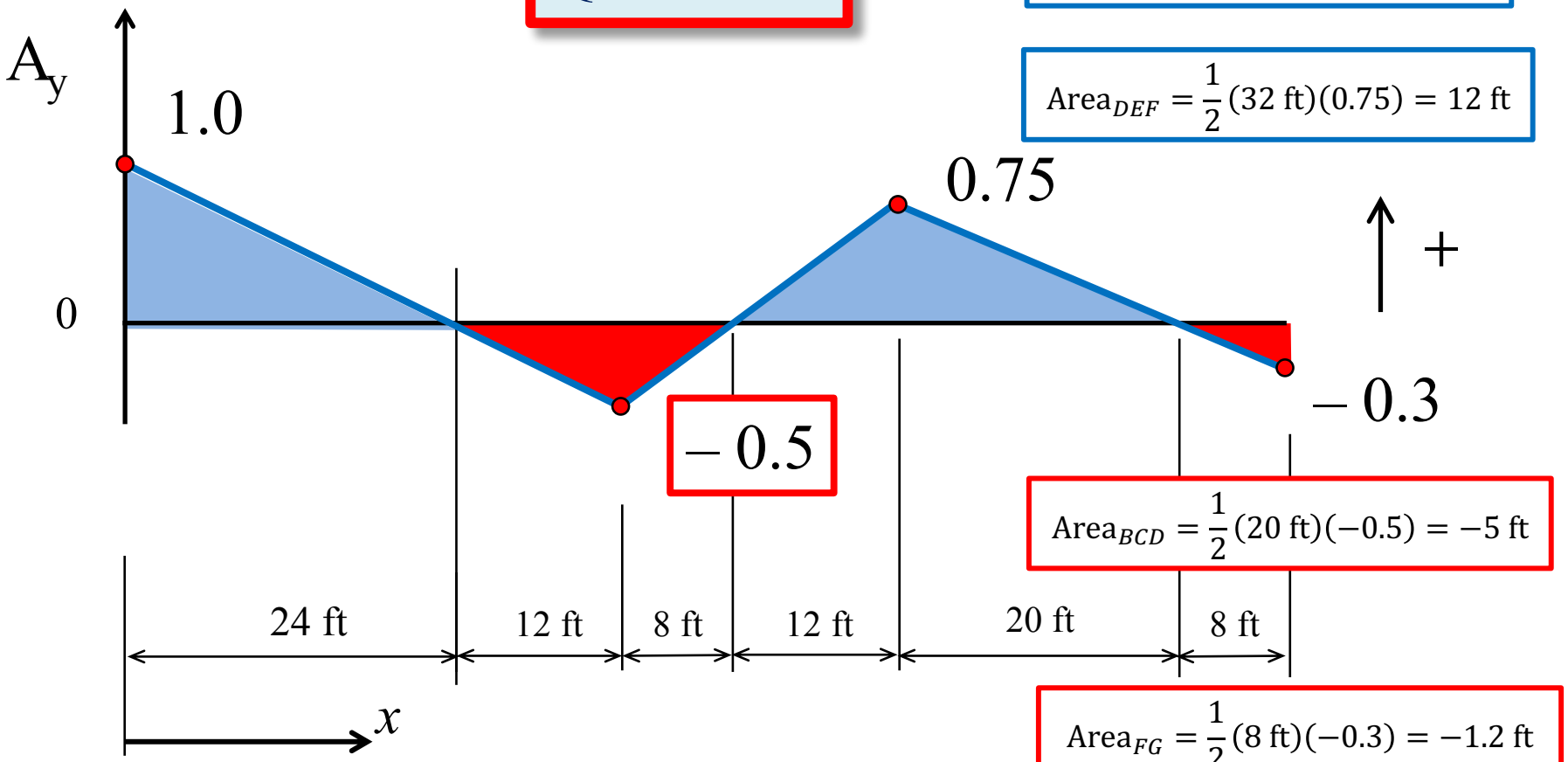
Dead load is fixed, so A_{yD} remains the same.

$A^{-}_{ymax} = 26.7\text{ k} - 34.1\text{ k} = -7.4\text{k}$

Question 4


$P_L = 90$ k
$w_D = 1.5$ k/ft
A
B
C
D
E
F
G
24 ft
12 ft
8 ft
12 ft
20 ft
8 ft
x
Place the point live load over the maximum negative ordinate of the A_y influence line to find $A_y^-{}_{max}$
A_y
1.0
0
0.75
– 0.5
– 0.3
+

Question 4



$\text{Area}_{AB} = \frac{1}{2}(24\text{ ft})(1.0) = 12\text{ ft}$

$\text{Area}_{DEF} = \frac{1}{2}(32\text{ ft})(0.75) = 12\text{ ft}$

$\text{Area}_{BCD} = \frac{1}{2}(20\text{ ft})(-0.5) = -5\text{ ft}$

$\text{Area}_{FG} = \frac{1}{2}(8\text{ ft})(-0.3) = -1.2\text{ ft}$

$A^{-}_{ymax_L} = (90\text{ k})[-0.5] = -45\text{ k}$

$A_{y_D} = 26.7\text{ k}$

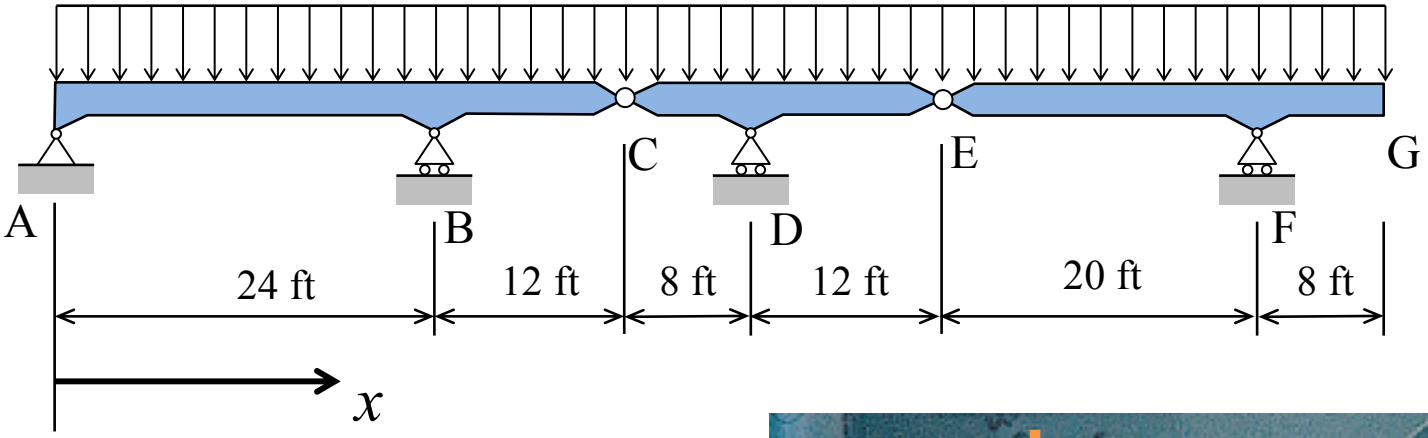
Dead load is fixed, so A_{yD} remains the same.

$A^{-}_{ymax} = 26.7\text{ k} - 45\text{ k} = -19.3\text{ k}$

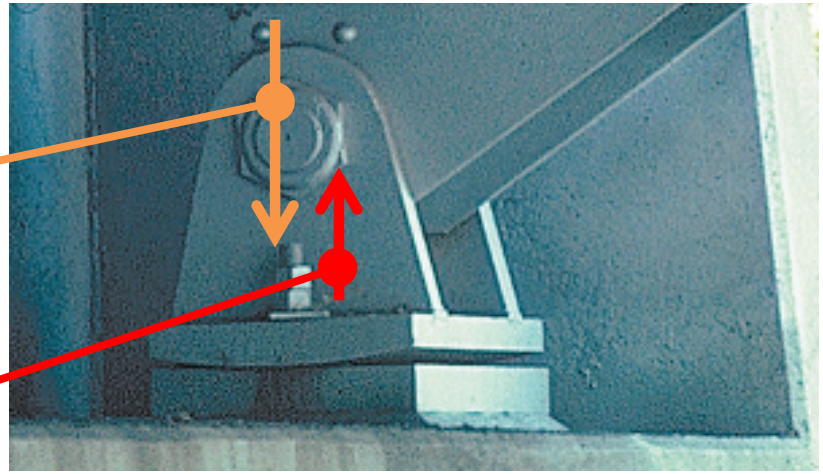
Summary of Results for A_{ymax}

$P_L = 90$ k **OR** $w_L = 5.5$ k/ft

$w_D = 1.5$ k/ft



L	A_{ymax}
5.5 k/ft	**158.7 k**
90 k	116.7 k
5.5 k/ft	–7.4 k
90 k	**–19.3 k-ft**



Pin Support at Point A