Equations of Equilibrium

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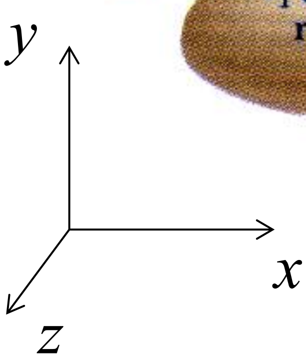
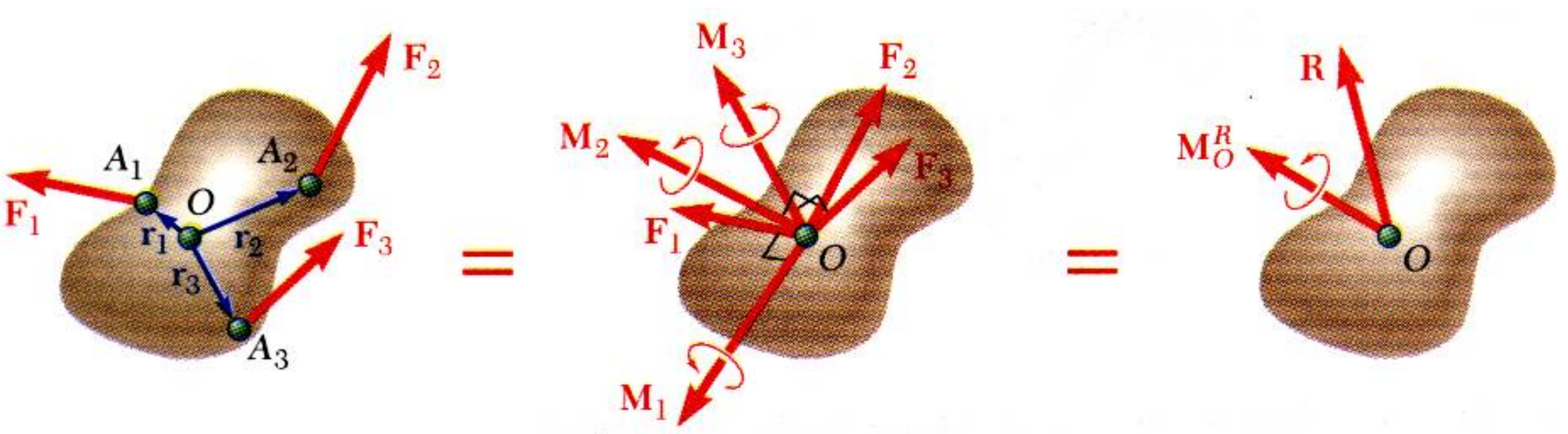
Recall that any Force System Acting on a Body can be Expressed as an Equivalent Force-Couple System at any Point





$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\vec{M}_O^R = \sum(\vec{r}\times\vec{F}) = \left(\sum M_x\right)\hat{\imath} + \left(\sum M_y\right)\hat{\jmath} + \left(\sum M_z\right)\hat{k}$$

A Body is in Equilibrium if Both the Resultant Force and Resultant Couple are Equal to Zero

Vector equations

$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_y\right)\hat{\jmath} = \vec{0}$$

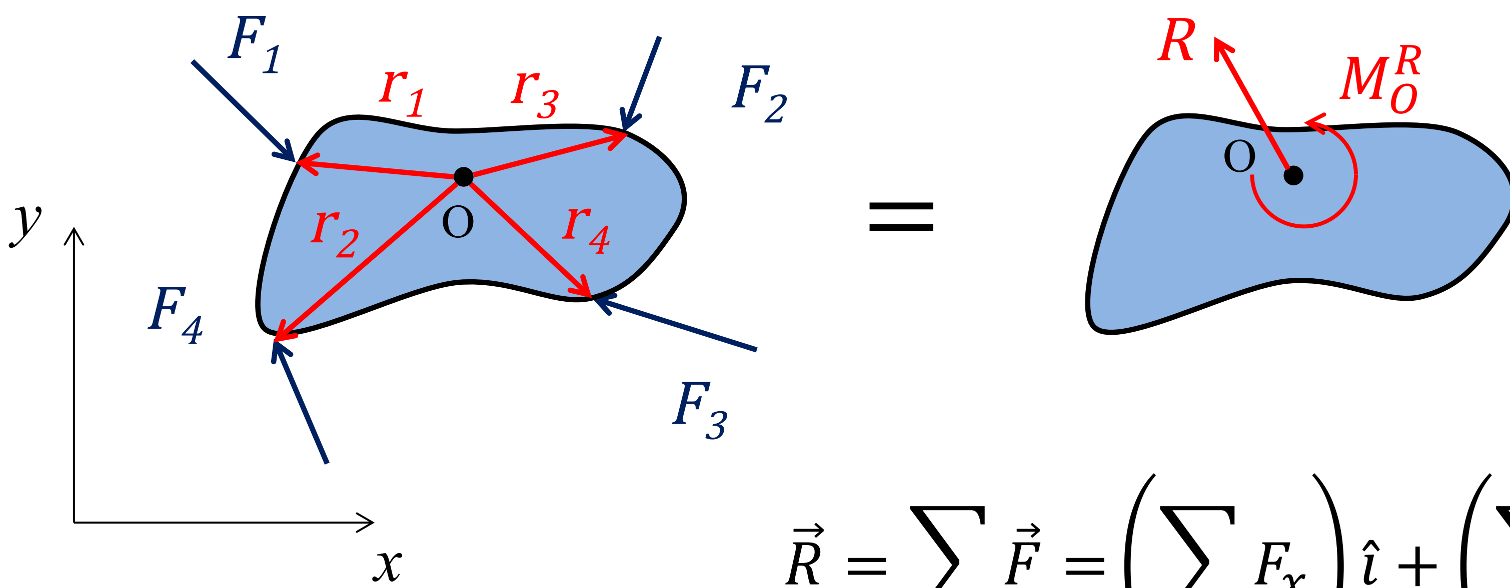
$$\vec{M}_O^R = \sum(\vec{r} \times \vec{F}) = \left(\sum M_x\right)\hat{\imath} + \left(\sum M_y\right)\hat{\jmath} + \left(\sum M_z\right)\hat{k} = \vec{0}$$

Six scalar equations of equilibrium

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

For a Two-Dimensional (Planar) Body in the *xy* Plane, the Resultant Moment Vector will Always be in the *z* Direction



$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\vec{M}_O^R = \left(\sum (\vec{r} \times \vec{F})\right)\hat{k}$$

For a General Two-Dimensional Body, the Six Scalar Equations Simplify to Three

Vector equations

$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\vec{M}_O^R = \left(\sum (\vec{r} \times \vec{F})\right)\hat{k}$$

Three scalar equations of equilibrium

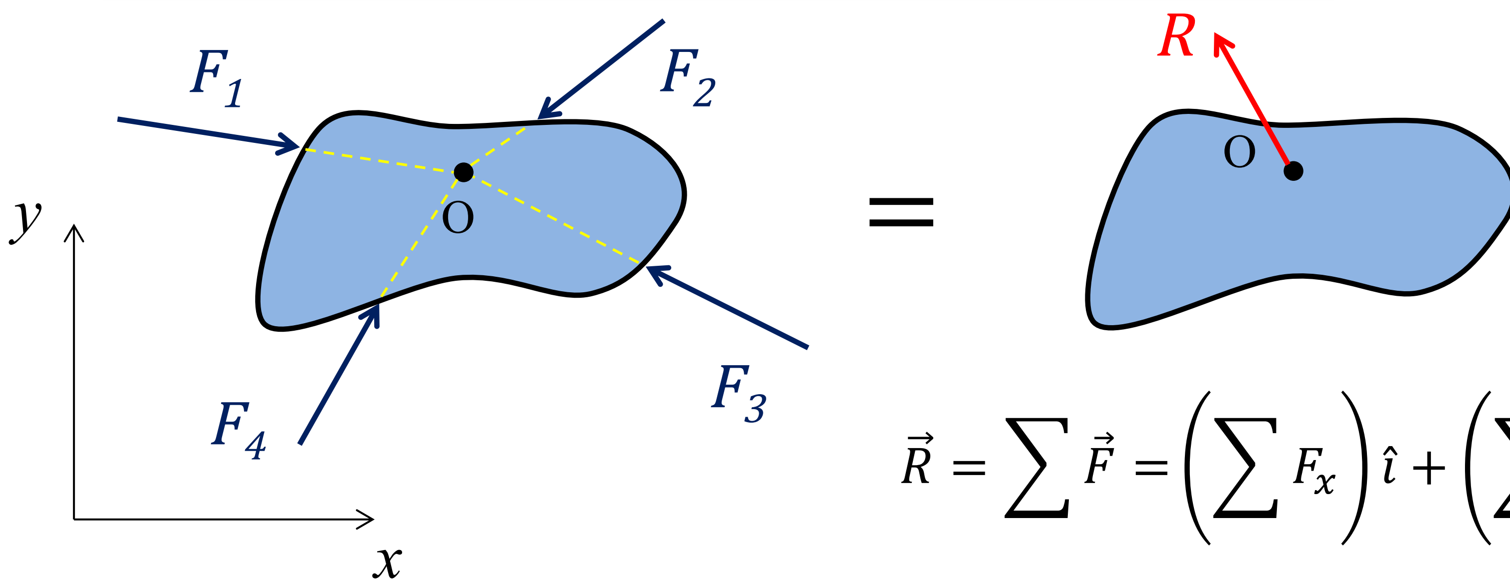
$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\sum M_z = 0$$

Can take moments about any point

For a body where all of the forces are concurrent, rotational equilibrium is satisfied

Planar body with concurrent forces



$$\vec{R} = \sum \vec{F} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} = \vec{0}$$

Two scalar equations of equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0$$

General Procedure for the Analysis of Bodies in Static Equilibrium

- Choose the free body to isolate;
- Draw a **Free Body Diagram (FBD)** of the body;
  - Isolate the body from all of its surroundings,
  - Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated,
  - Indicate dimensions on the FBD,
- Write the **equations of equilibrium** and solve the equations for the unknown quantities.