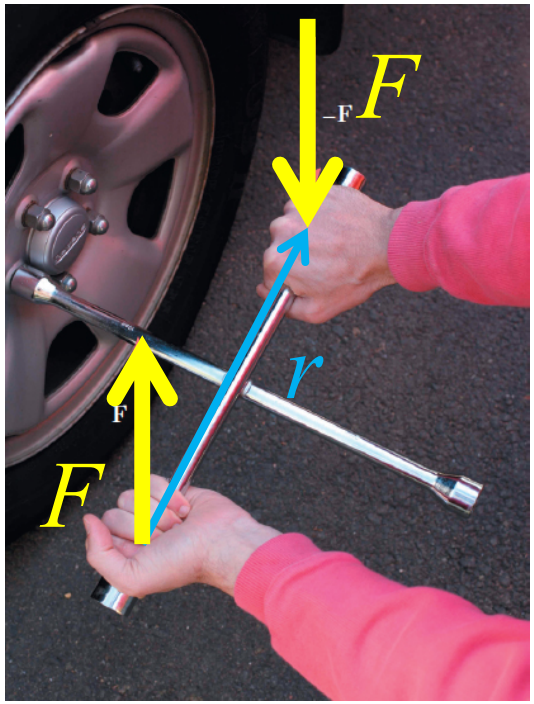
Force-Couple System

Steven Vukazich
San Jose State University

Moment of A Force About a Point

In order to analyze structures subjected to general force systems that can cause rotation, we introduced the concept of the moment of a force about a point

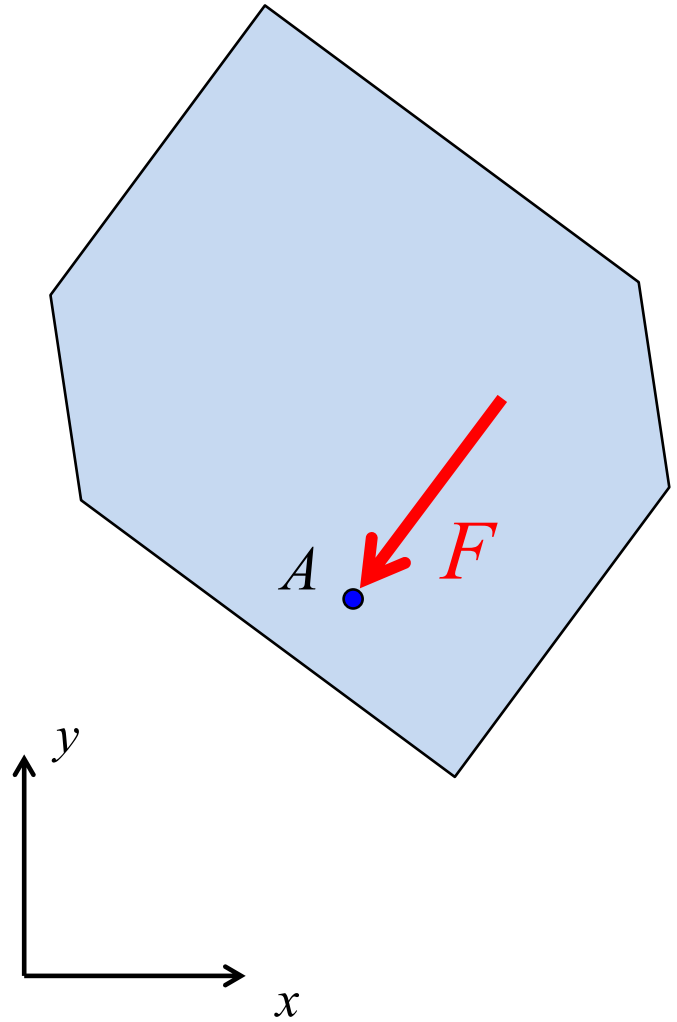
Moment of a Couple



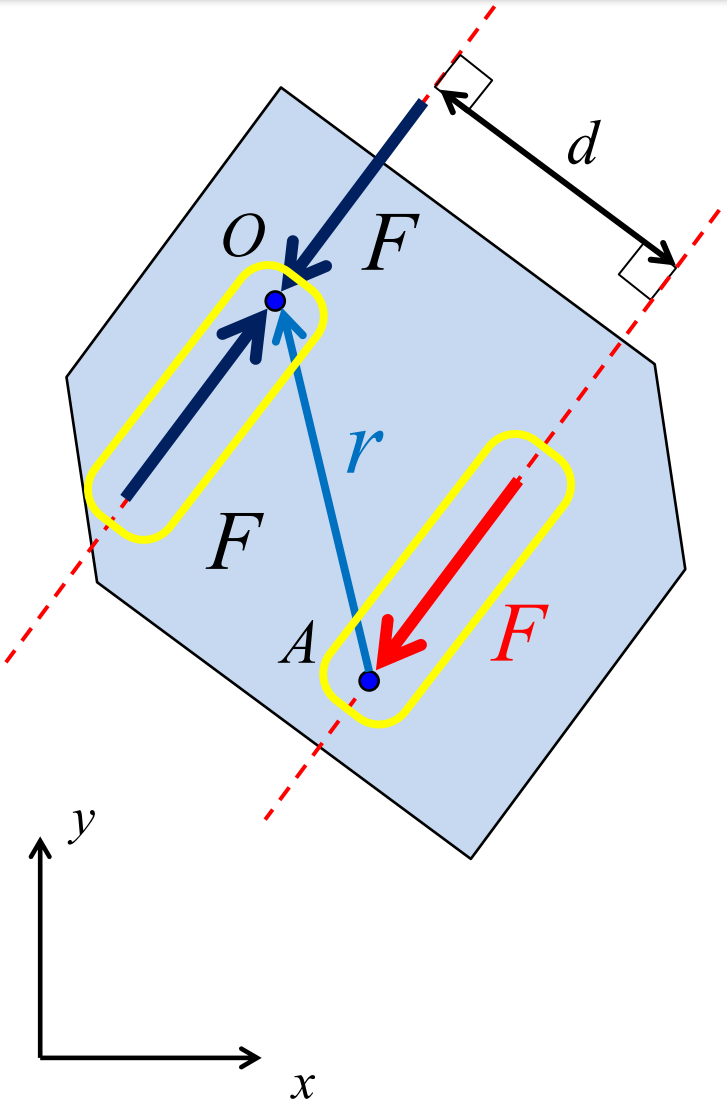
We then added the related concept of the moment of a couple

We are now ready to understand one of the fundamental concepts for the analysis of bodies subjected to general force systems – the force-couple system

Consider a force acting on a Planar Body at Point A



At Point *O*, add Two Forces With the Same Magnitude, Same Line of Action, and Opposite Sense



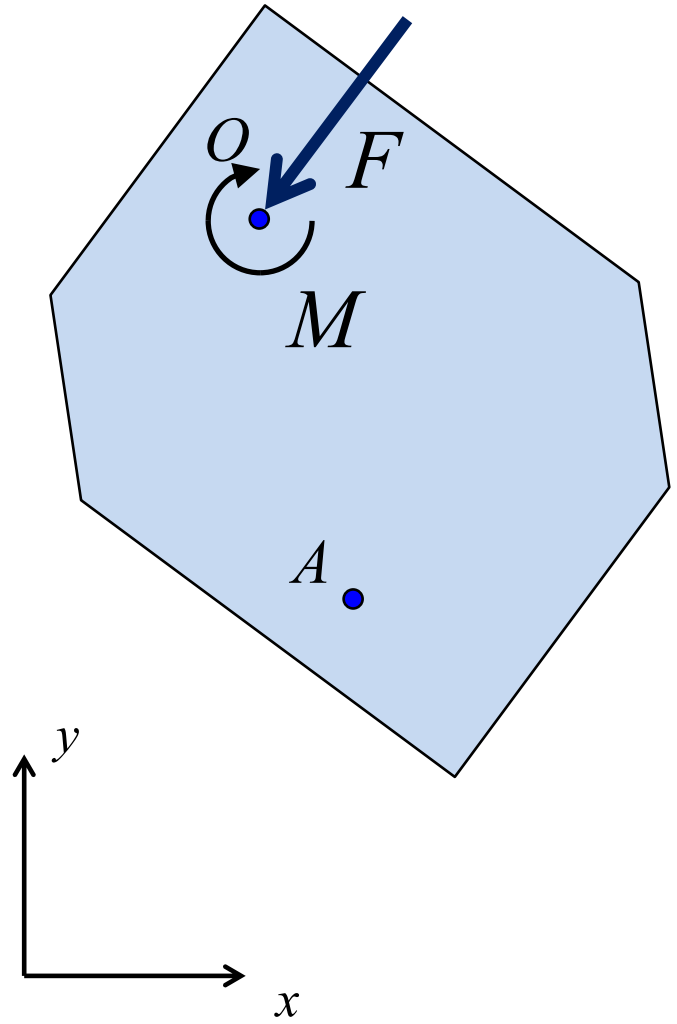
The net effect of the forces at point ***O*** on the body is zero

The two forces outlined in yellow form a couple

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$$

$$M = dF$$

Equivalent Force-Couple System

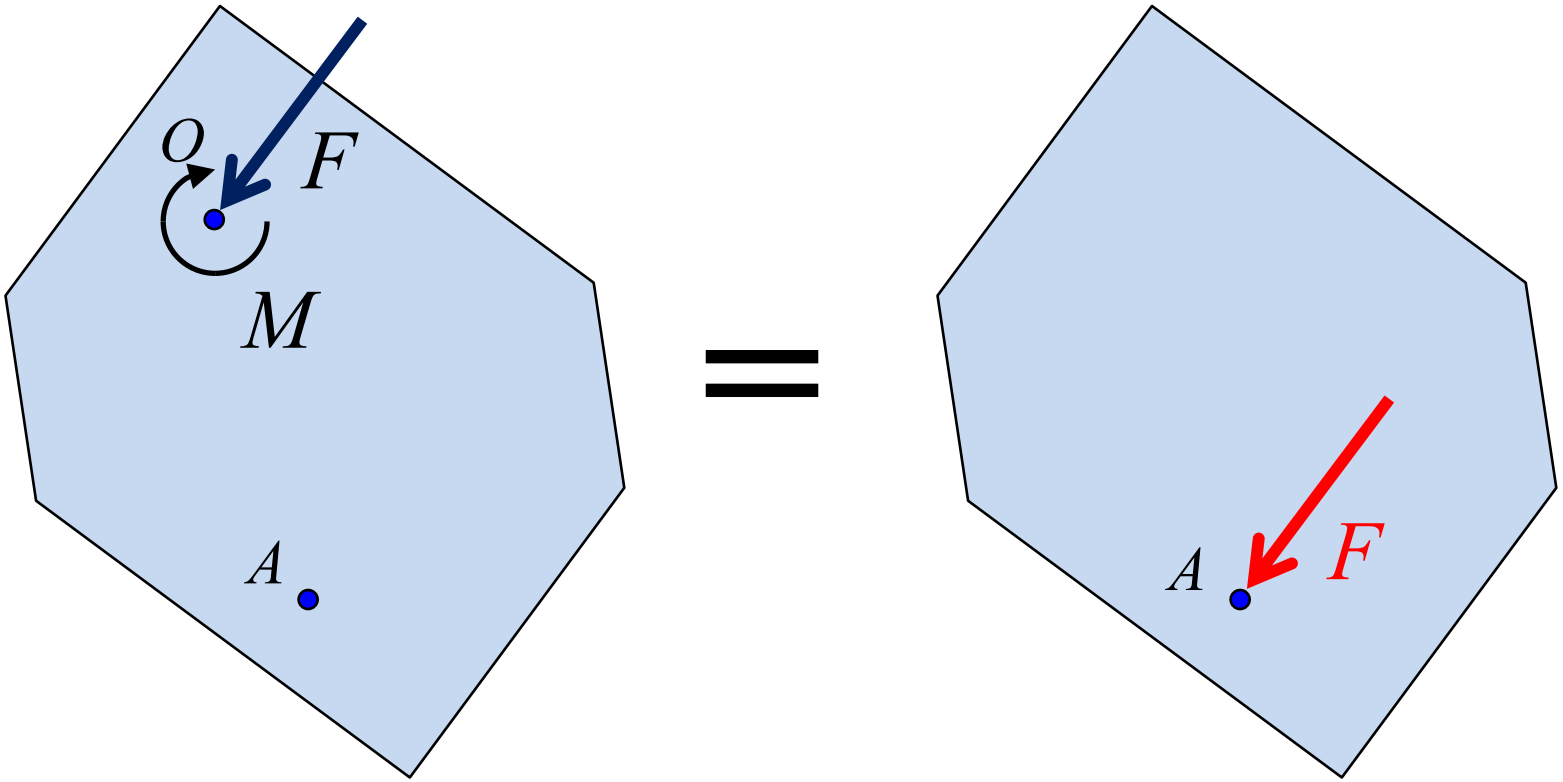


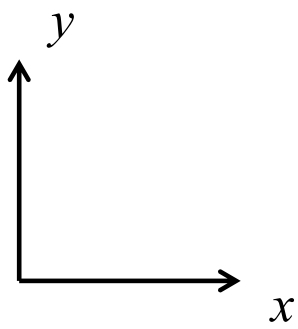
The force-couple system at point ***O*** has same effect on the body as the force applied at ***A***!

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$$

$$M = dF$$

Equivalent Force-Couple System

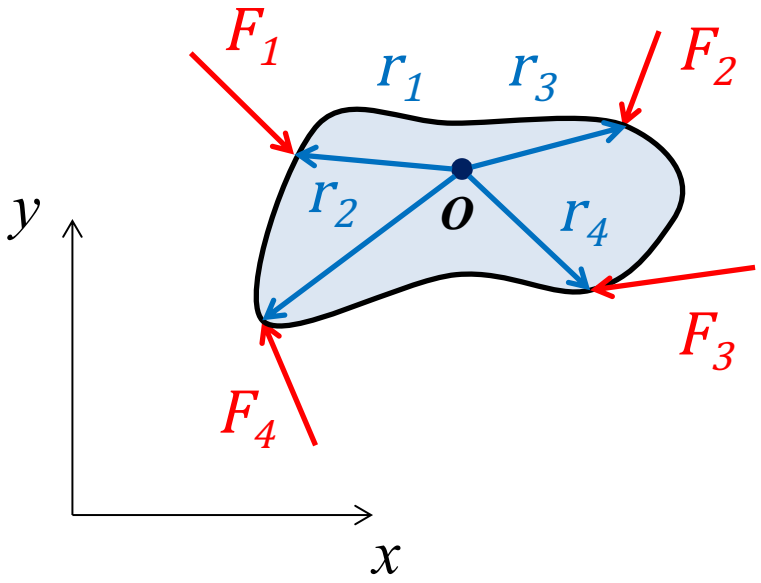




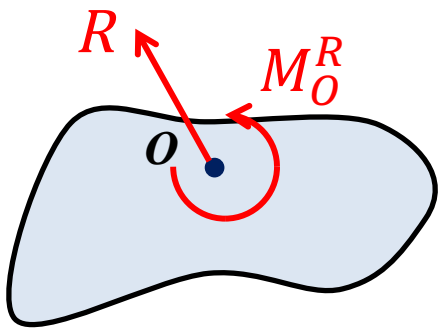
Note that the couple in the force-couple system is equal to the moment of the force ***F*** about point ***O***

Any Force System Acting on a Rigid Body can be Replaced by a Resultant Force-Couple System at Any Point

Planar Force System



=



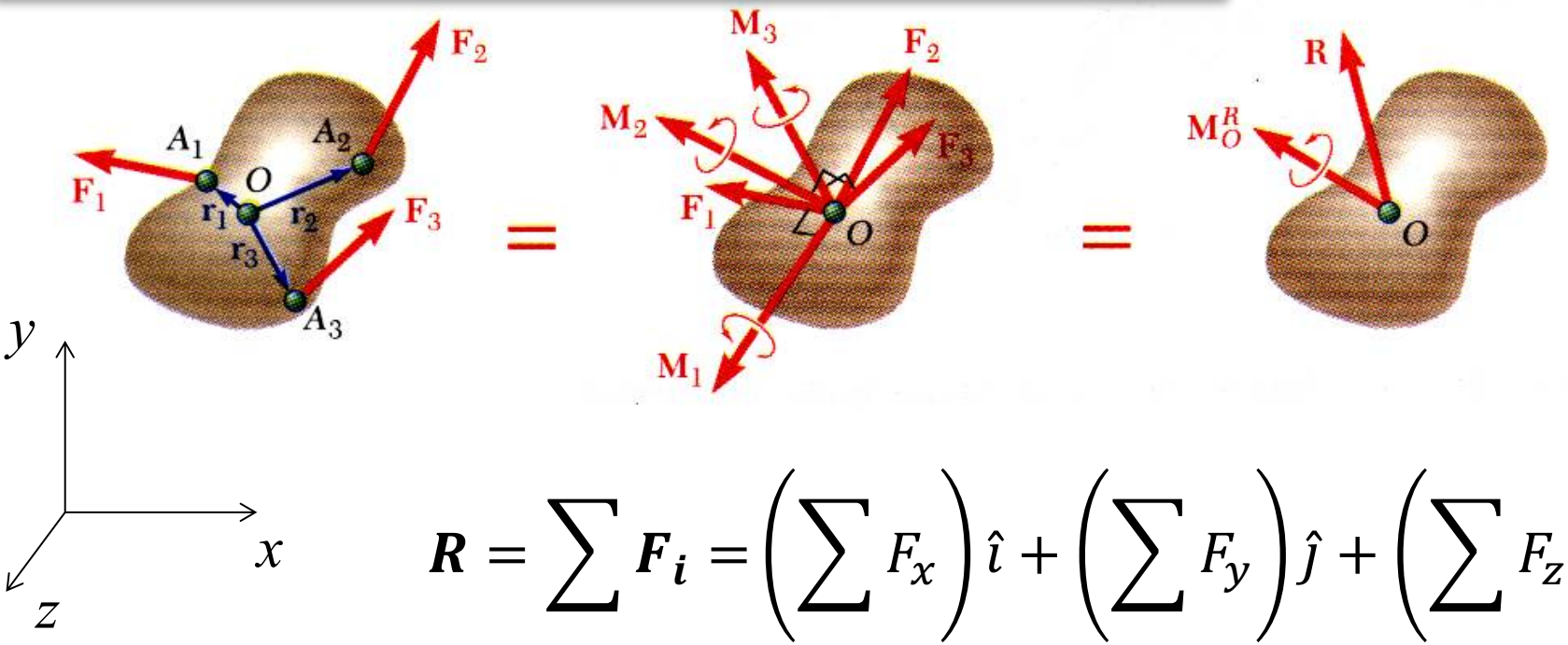
Note that the resultant force is always the same but the resultant couple depends on the location of the point

$$\boldsymbol{R} = \sum \boldsymbol{F_i} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath}$$

$$\boldsymbol{M_O^R} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i})$$

Any Force System Acting on a Rigid Body can be Replaced by a Resultant Force-Couple System at Any Point

General Three-Dimensional Force System



$$\boldsymbol{R} = \sum \boldsymbol{F_i} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_z\right)\hat{k}$$

$$\boldsymbol{M_O^R} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i})$$