Shear and Bending Moment Diagrams for Beams

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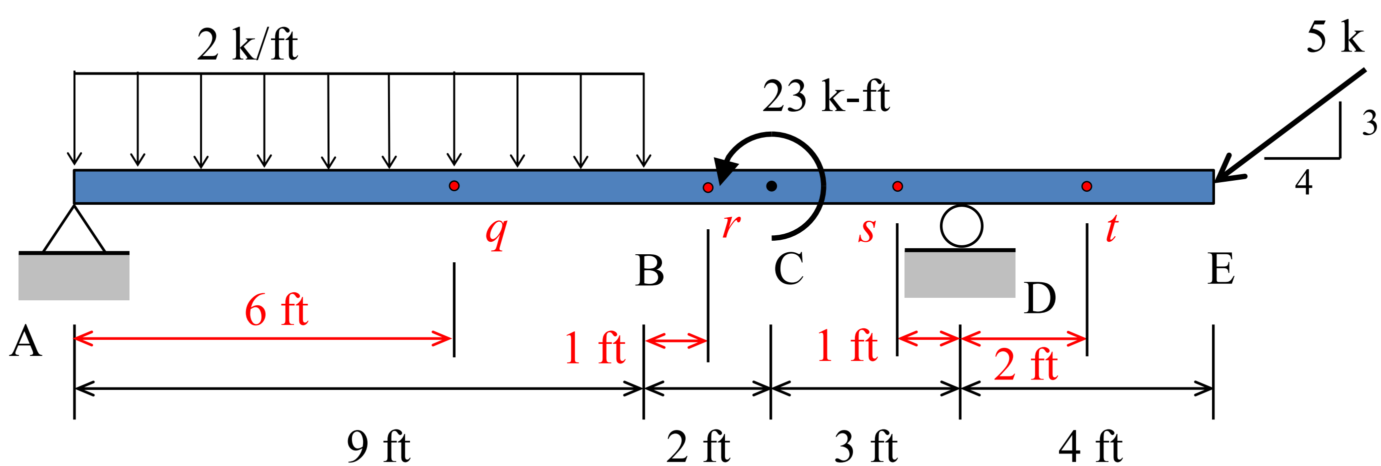
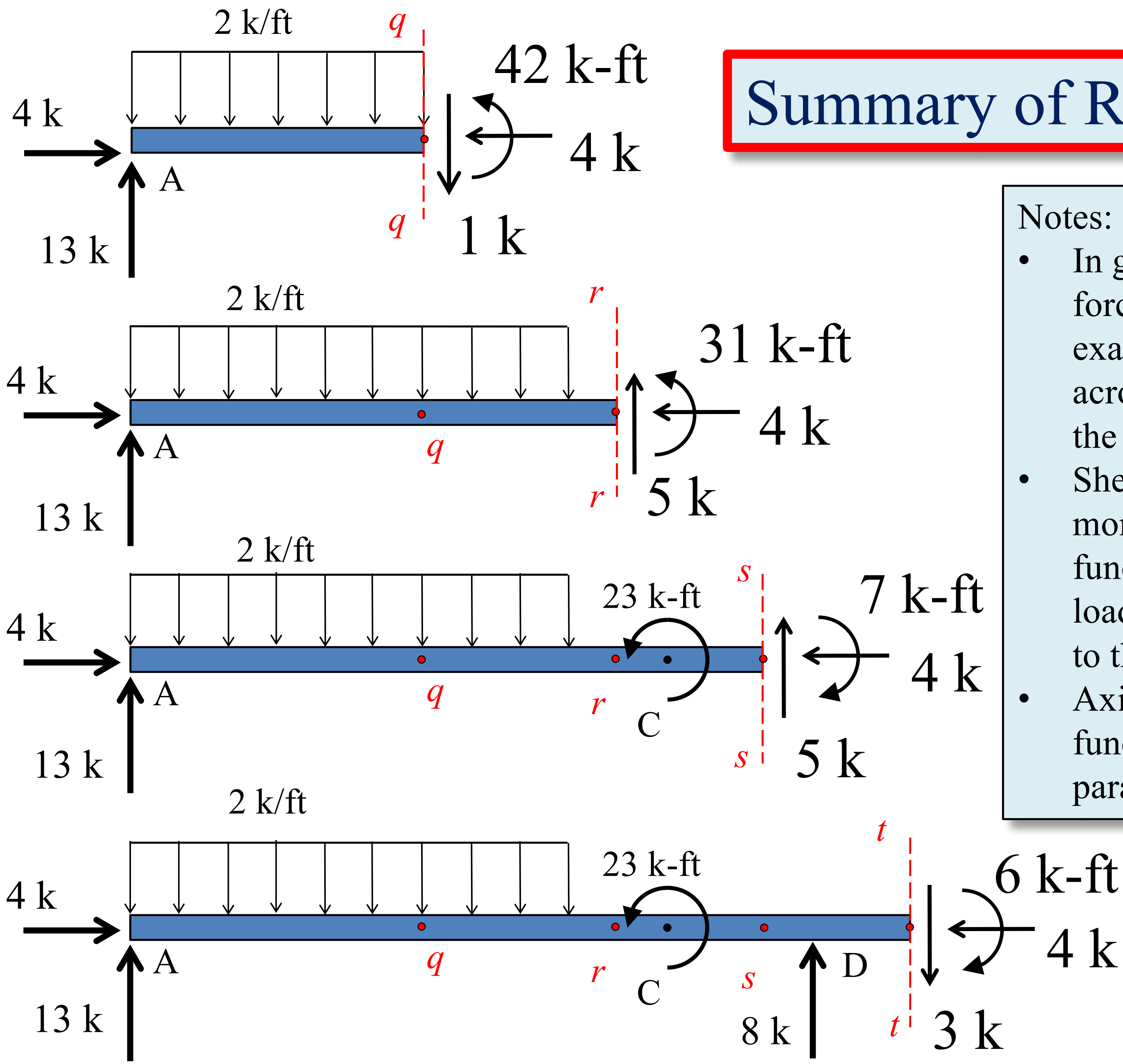
Recall the Internal Force Example Problem

A beam is supported by a pin support at point A and extends over a roller support at point D. The beam is and subjected to a uniformly distributed load from A to B, a point moment at point C an inclined point load at point E as shown.

Find the internal forces at points *q*, *r*, *s*, and *t*.



Summary of Results



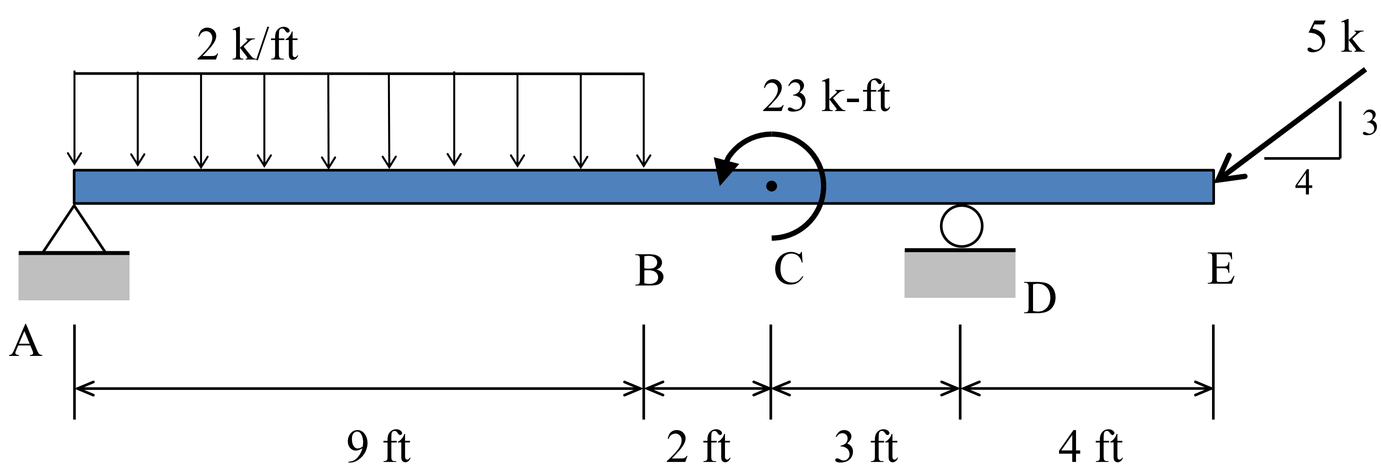
Notes:

- In general, internal forces vary as we examine points across the length of the beam;
- Shear and bending moment are both functions of the loads perpendicular to the beam;
- Axial force is a function of loads parallel to the beam.

Find Shear and Bending Moment Diagrams

A beam is supported by a pin support at point A and extends over a roller support at point D. The beam is and subjected to a uniformly distributed load from A to B, a point moment at point C an inclined point load at point E as shown.

Find the internal shear (V) diagram and the internal bending moment diagram (M) for the beam.

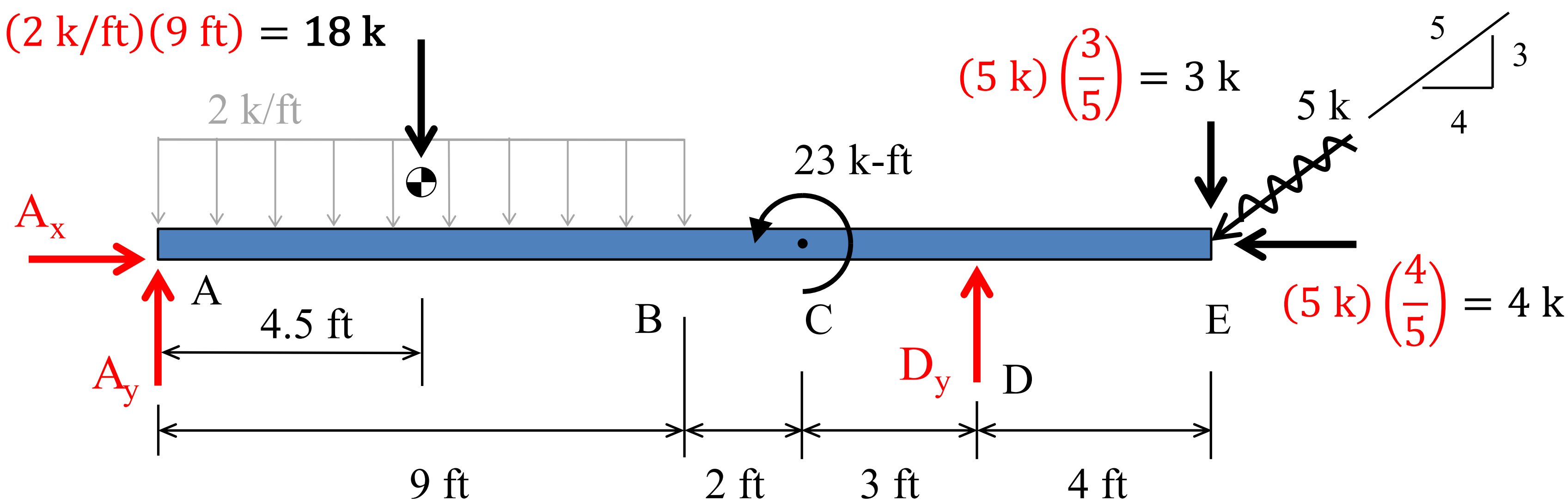


General procedure for the Construction of Shear and Bending Moment Diagrams

1. Find all of the external forces (support reactions);
2. Choose a sign convention for each diagram;
3. Choose a reference coordinate system:
4. Use equilibrium analysis to construct internal force functions for each relevant section of the beam;
   - Cut beam at appropriate sections,
   - The FBD on either side of the cut may be analyzed,
   - Indicate unknown internal forces consistent with the chosen sign convention,
   - Plot the internal force function for each section,
5. Check each diagram for errors;
   - Check discontinuities at location of applied forces in shear diagram,
   - Check discontinuities at location of applied moment in moment diagram.

1. Find All of the External Forces

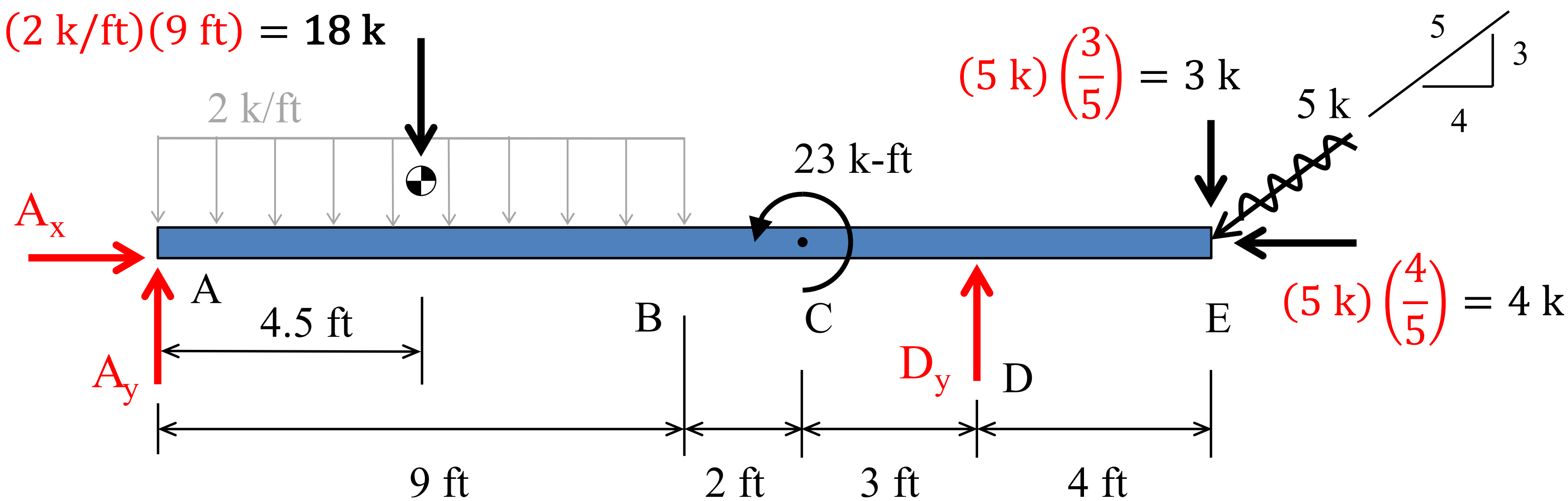
FBD of beam



$$\circlearrowleft + \sum M_A = 0$$

$D_y = 10\ \mathrm{k}$

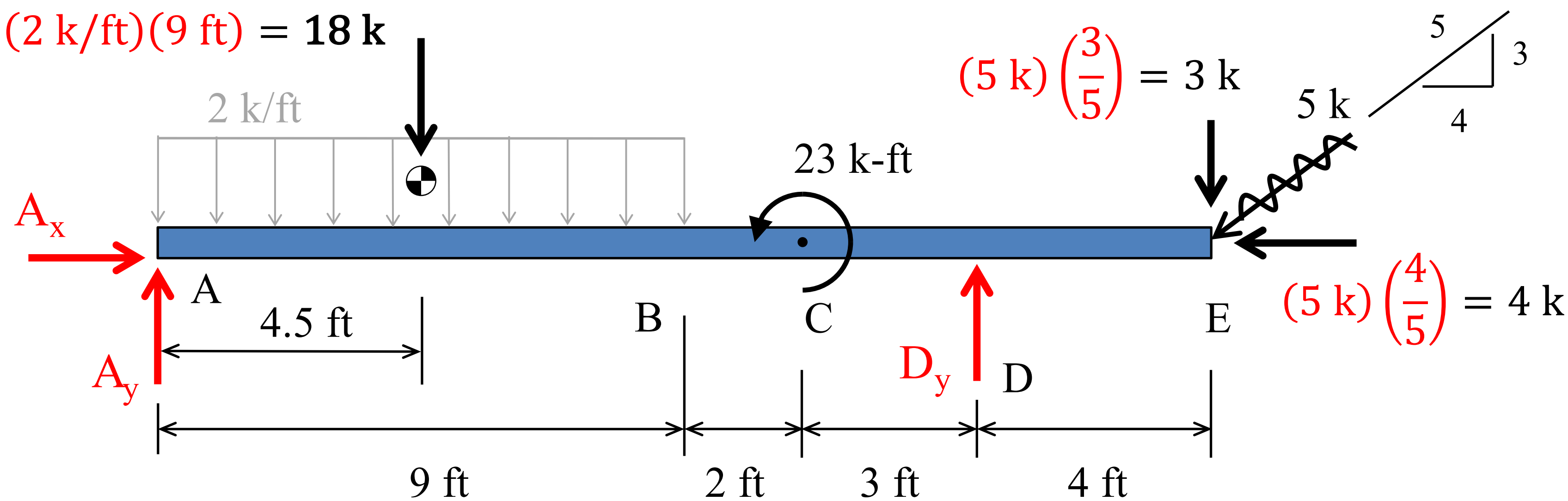
1. Find All of the External Forces



$$+\uparrow \sum F_y = 0$$

$A_y = 13$ k

1. Find All of the External Forces

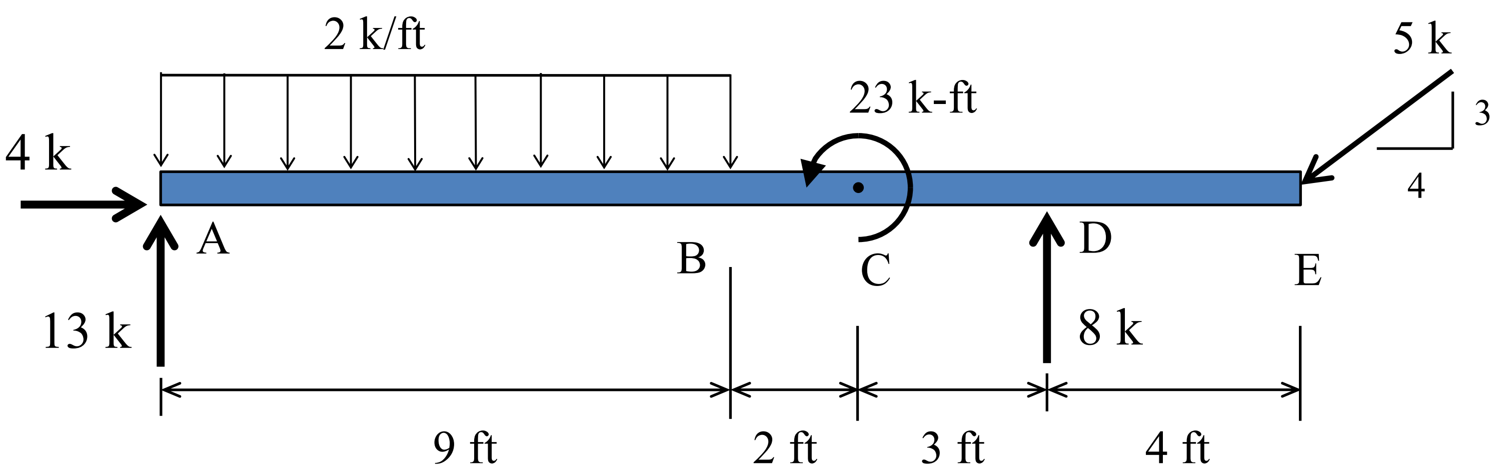


$$\xrightarrow{+} \sum F_x = 0$$

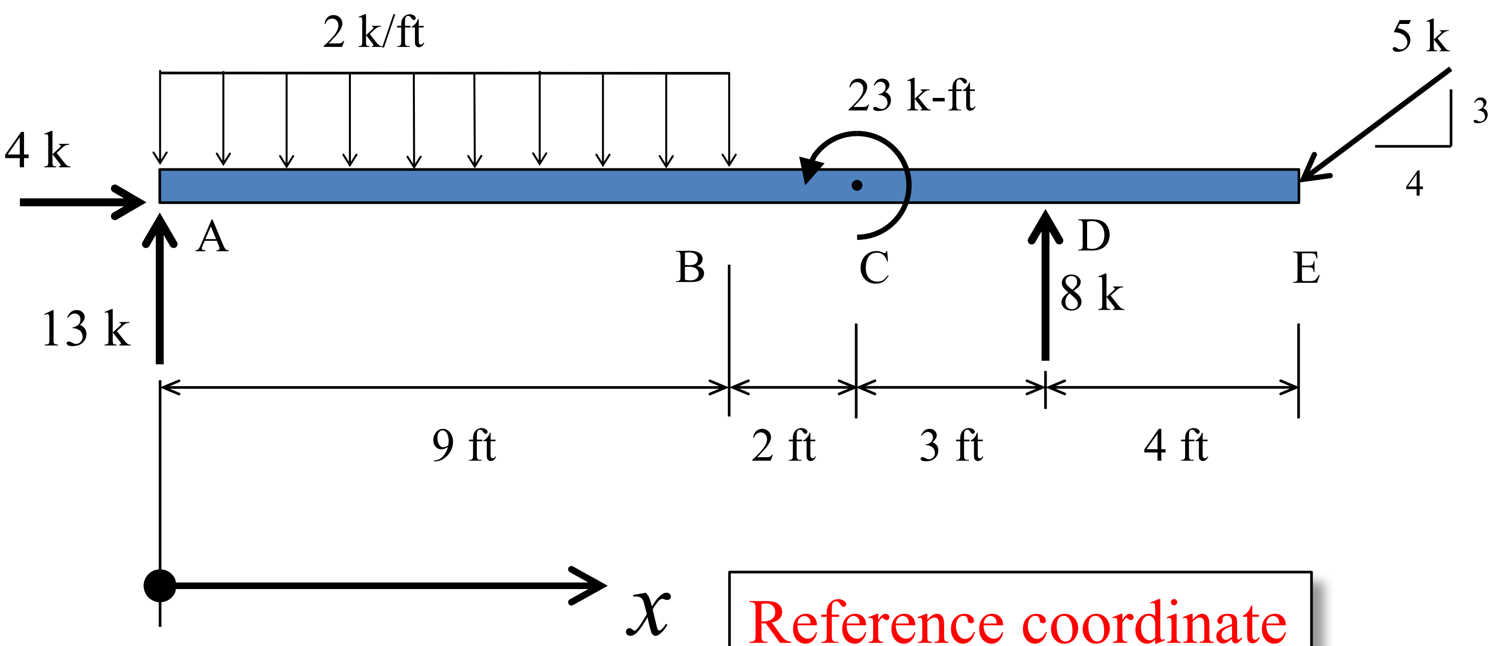
$A_x = 4$ k

External Forces

FBD of beam showing all external forces

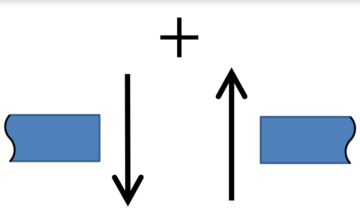


2. Choose a sign convention for each diagram;
3. Choose a reference coordinate system;

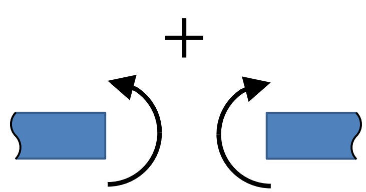


Reference coordinate

Postive shear

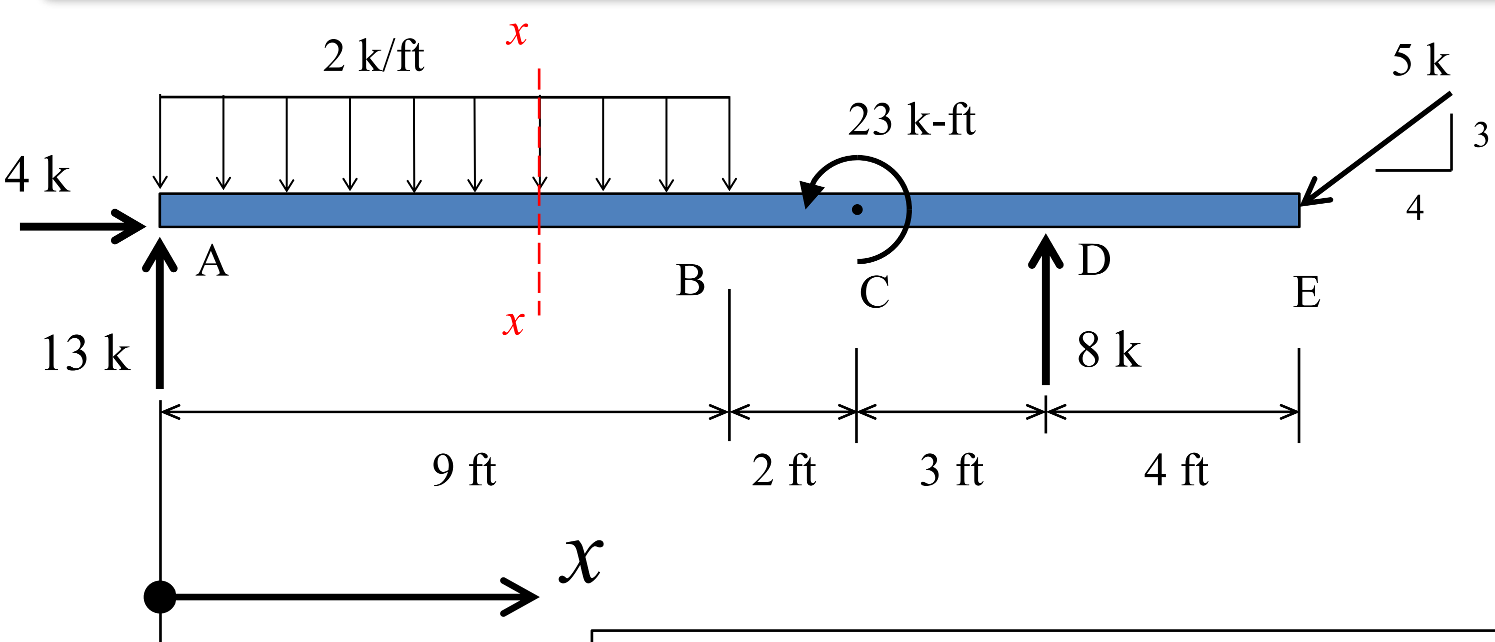


Positive bending moment



4. Use equilibrium analysis or differential and integral relationships to construct internal force functions;

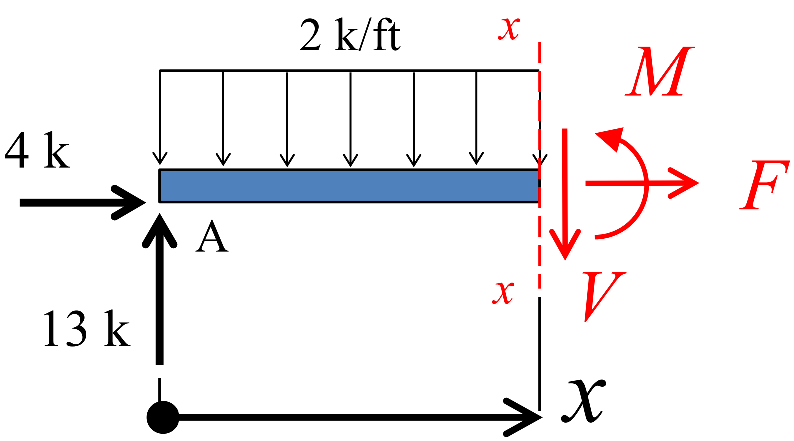
- Cut the beam at appropriate sections,
- The FBD on either side of the cut may be analyzed,
- Indicate unknown internal forces consistent with the chosen sign convention,
- Plot the shear and moment function for each segment,



First cut the beam at an arbitrary point between points A and B

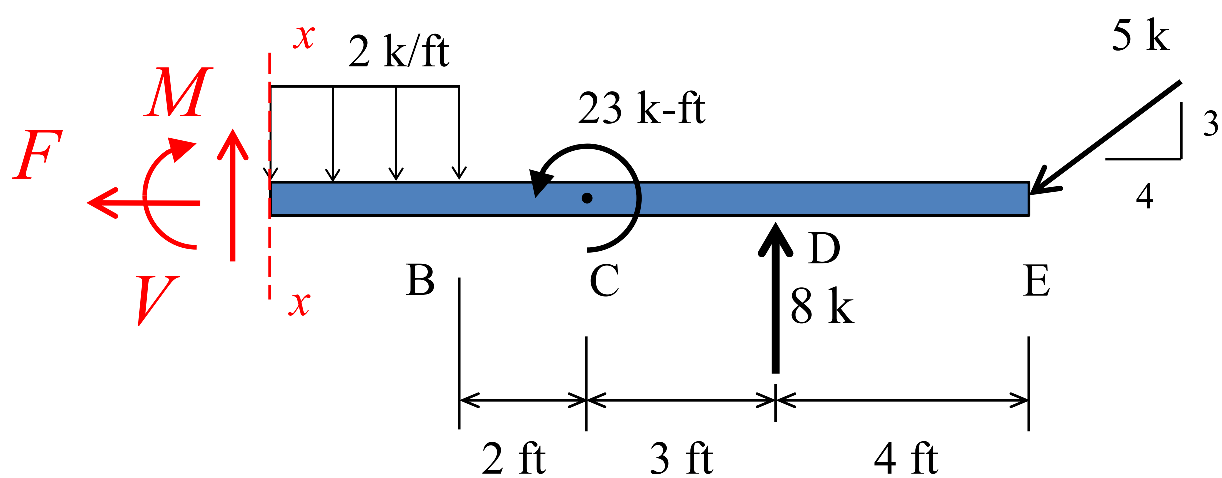
Find Internal Forces Between Points A and B

FBDs of Sections on either side of the cut



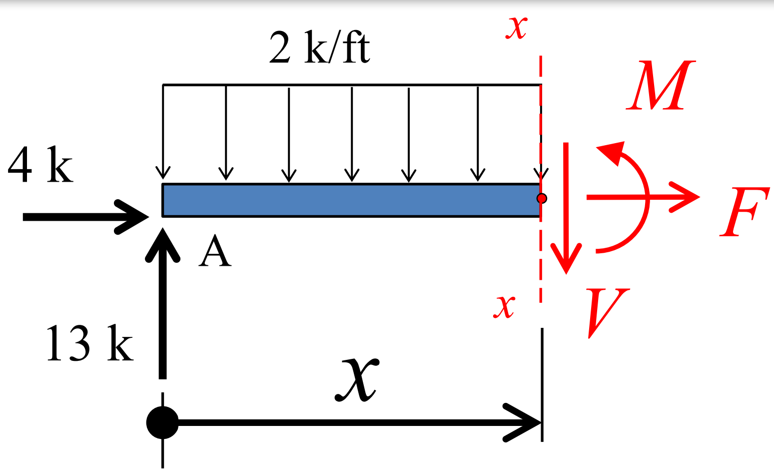
Can apply equations of equilibrium to either FBD to find internal forces

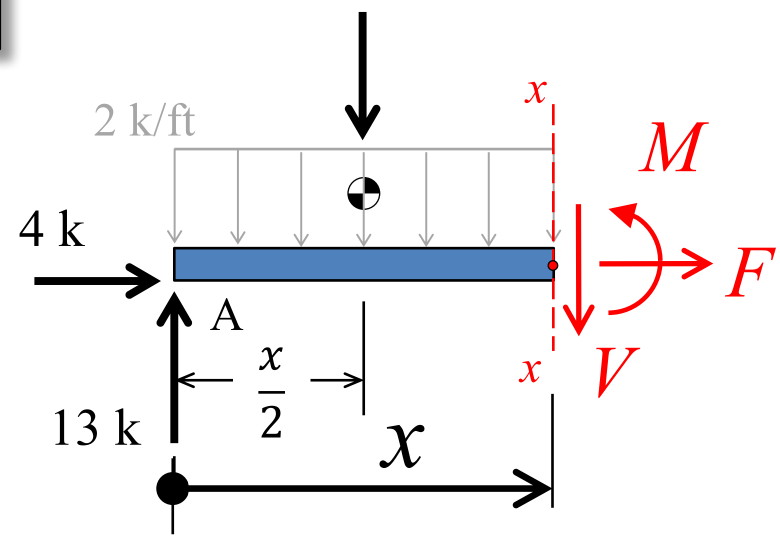
Section to the left of the cut is easiest to analyze



Find Internal Forces Between Points A and B

FBD of segment to the left of the cut





$$\xrightarrow{+} \sum F_x = 0 \qquad +\uparrow \sum F_y = 0$$

$$0 \leq x \leq 9$$

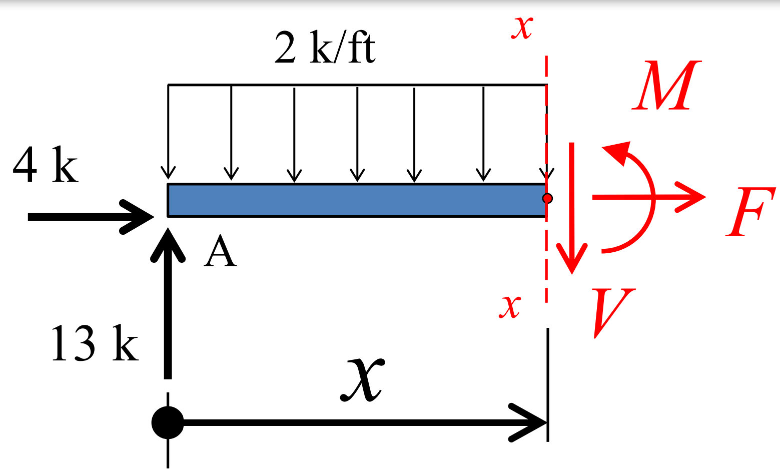
$$V(0) = V_A = 13\ \text{k}$$
$$\text{V}(9) = V_B = -5\ \text{k}$$

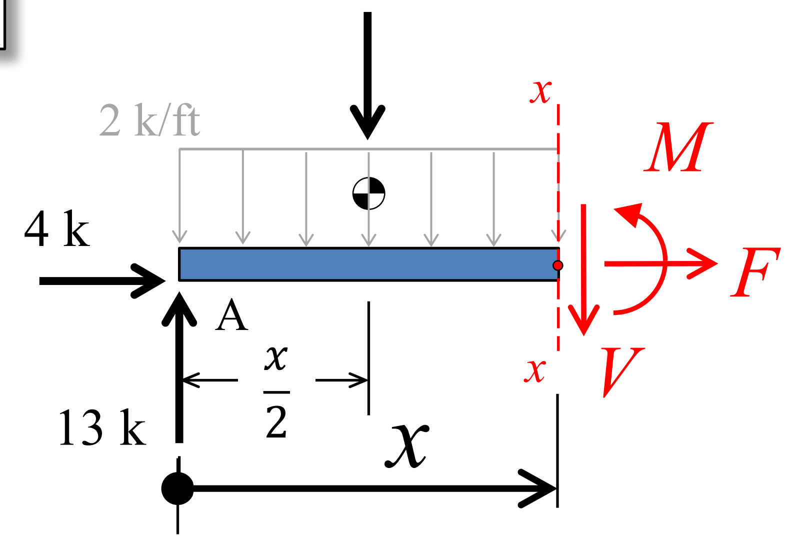
$$V = 13 - 2x$$

F = -4 k

Find Internal Forces Between Points A and B

FBD of segment to the left of the cut





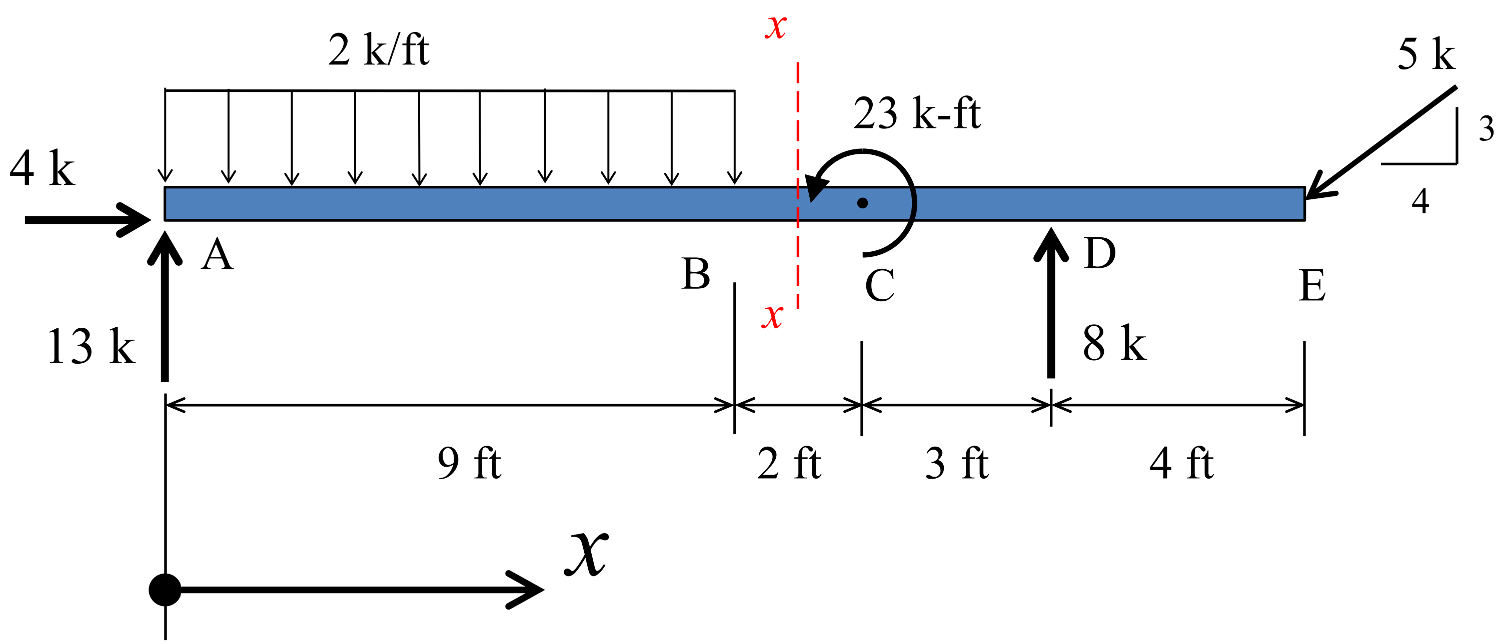
$$\circlearrowleft + \sum M_x = 0$$

$$0 \leq x \leq 9$$

$$M(0) = M_A = 0$$
$$M(9) = M_B = 36 \text{ k-ft}$$

$$M = 13x - x^2$$

Find Internal Forces Between Points B and C



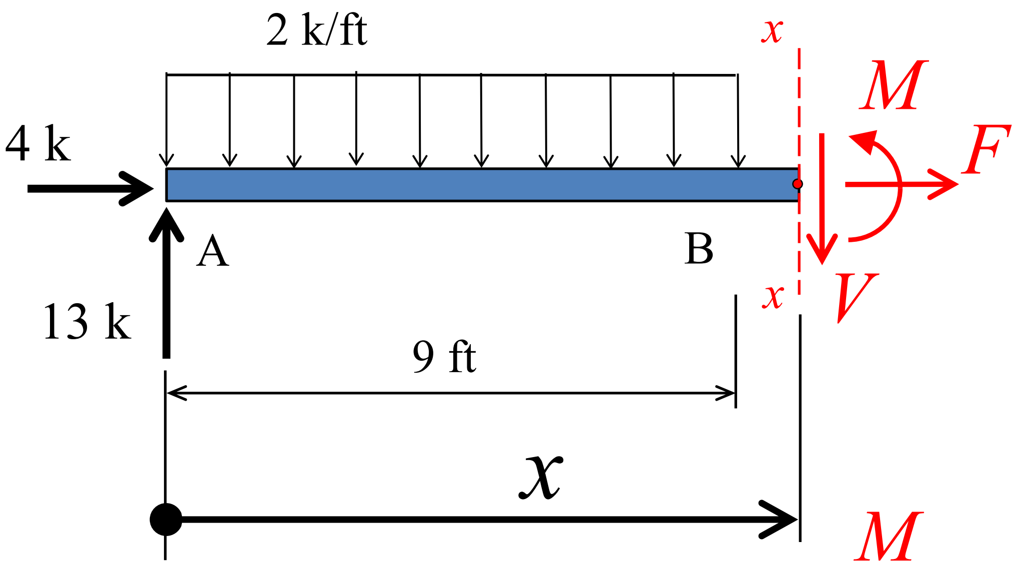
Next, cut the beam at an arbitrary point between points B and C

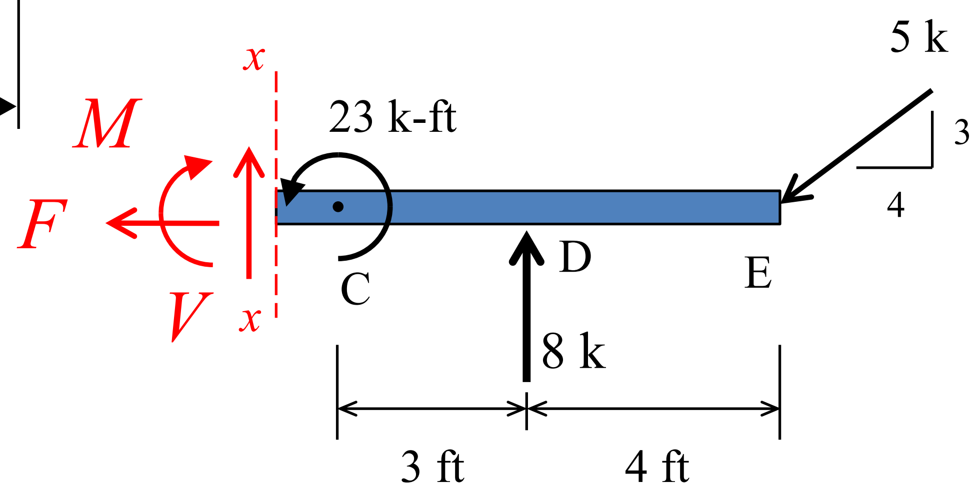
Find Internal Forces Between Points B and C

FBDs of Sections on either side of the cut

Can apply equations of equilibrium to either FBD to find internal forces

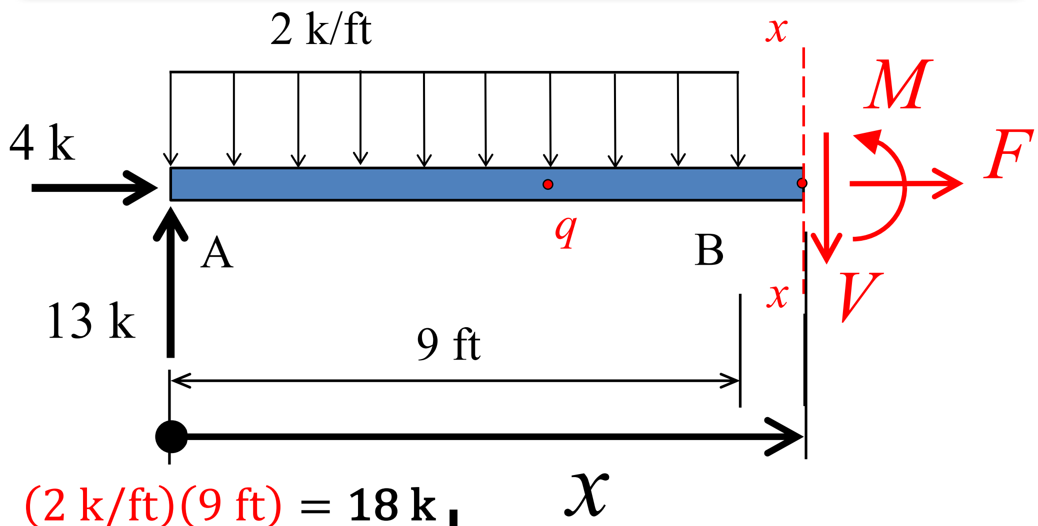
Let's continue to use the section to the left of the cut





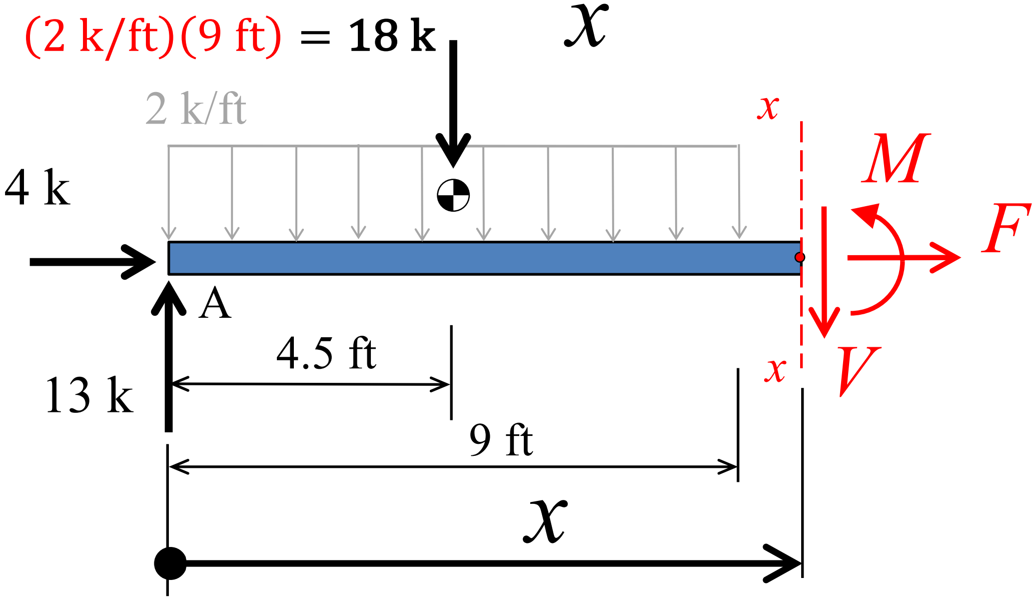
Find Internal Forces Between Points B and C

FBD of segment to the left of the cut



$$+\uparrow \sum F_y = 0$$

$$9 \leq x \leq 11$$
$$V(9) = V_B = -5 \text{ k}$$
$$\text{V}(11) = V_C = -5 \text{ k}$$



$$\circlearrowleft + \sum M_x = 0$$

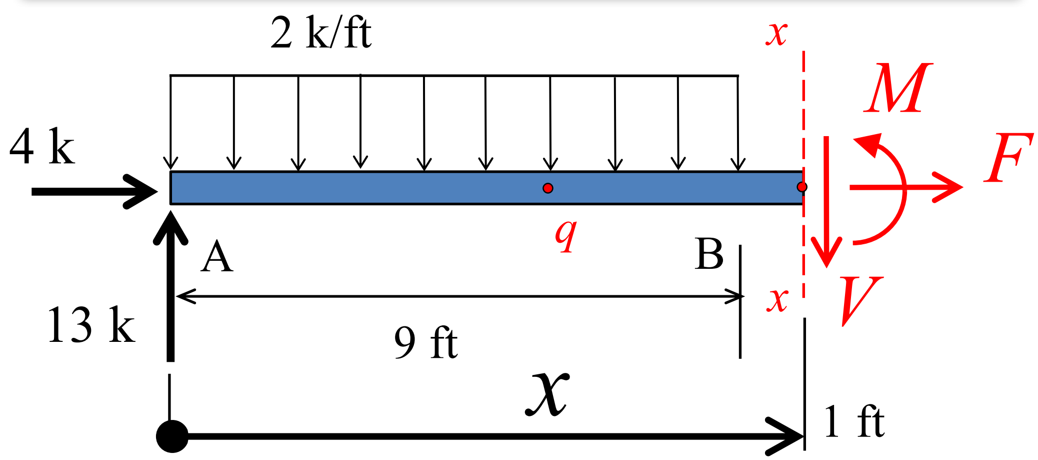
$$\xrightarrow{+} \sum F_x = 0$$

$$V = -5 \text{ k}$$

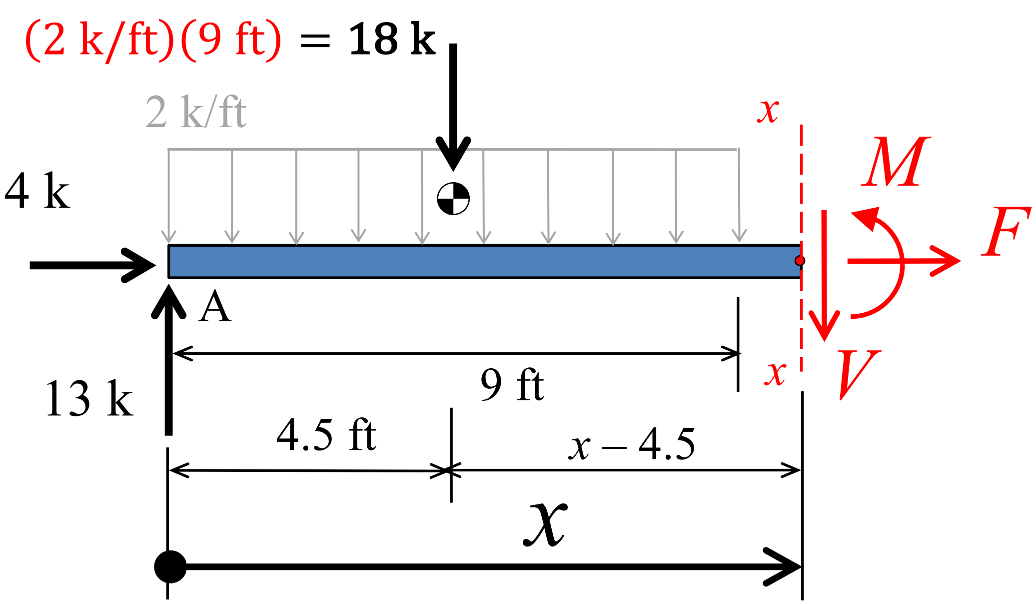
F = -4 k

Find Internal Forces Between Points B and C

FBD of segment to the left of the cut



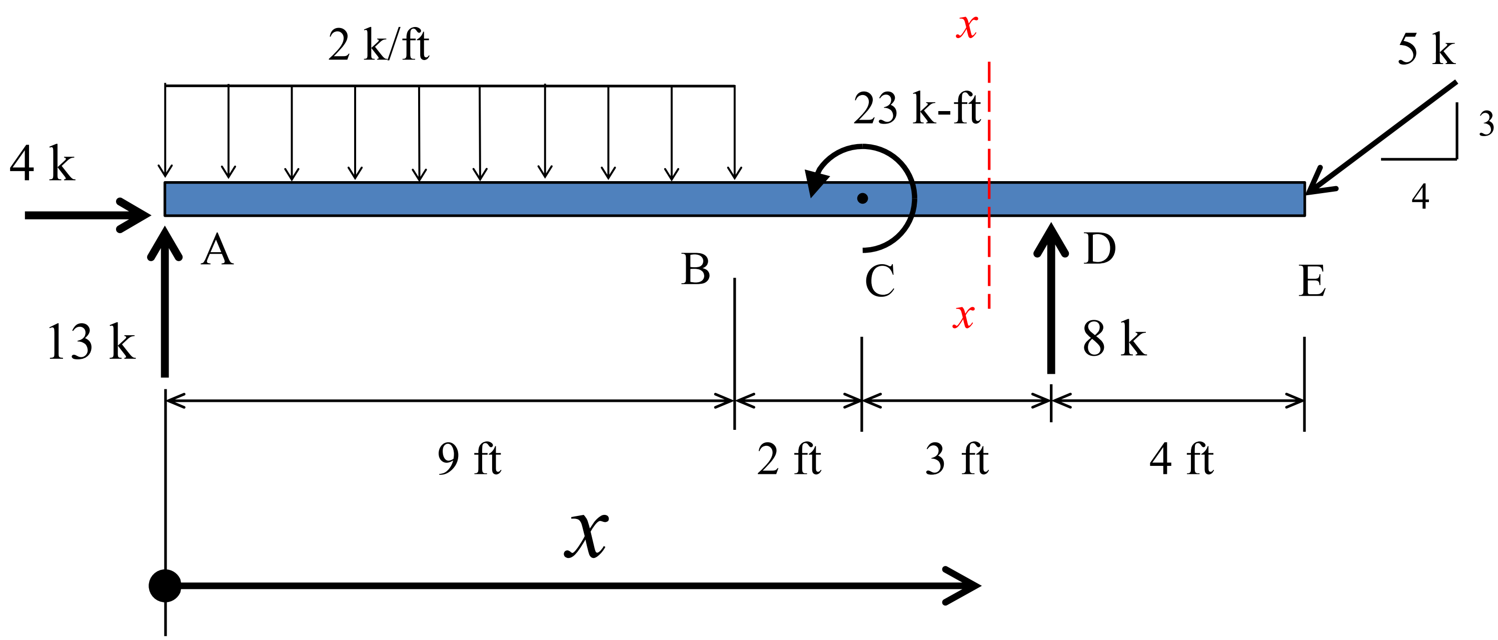
$$\circlearrowleft + \sum M_x = 0$$



$$9 \le x \le 11$$
$$M(9) = M_B = 36 \text{ k–ft}$$
$$M(11) = M_C = 26 \text{ k–ft}$$

$$M = -5x + 81$$

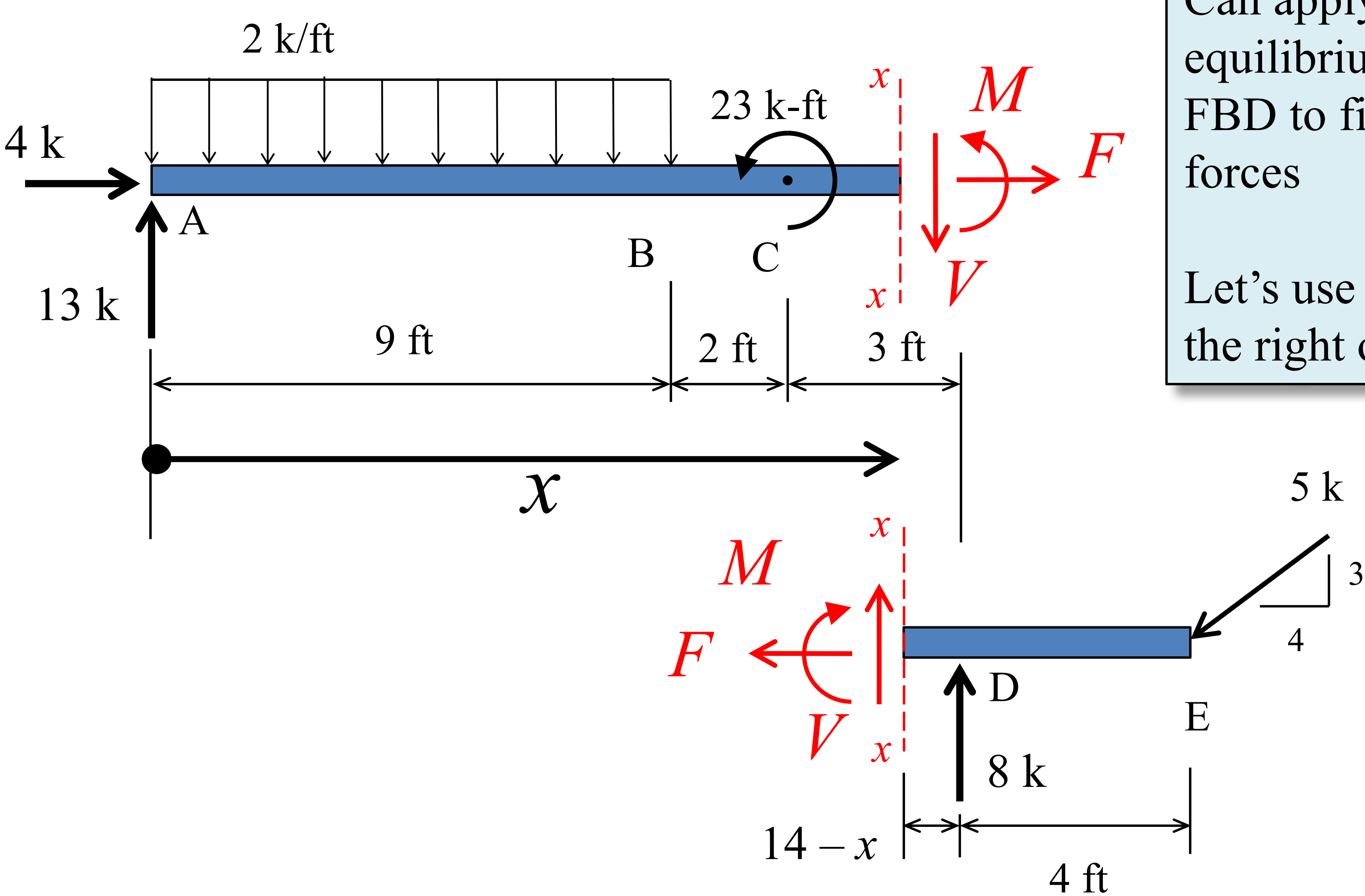
Find Internal Forces Between Points C and D



Next, cut the beam at an arbitrary point between points C and D

Find Internal Forces Between Points C and D

FBDs of Sections on either side of the cut

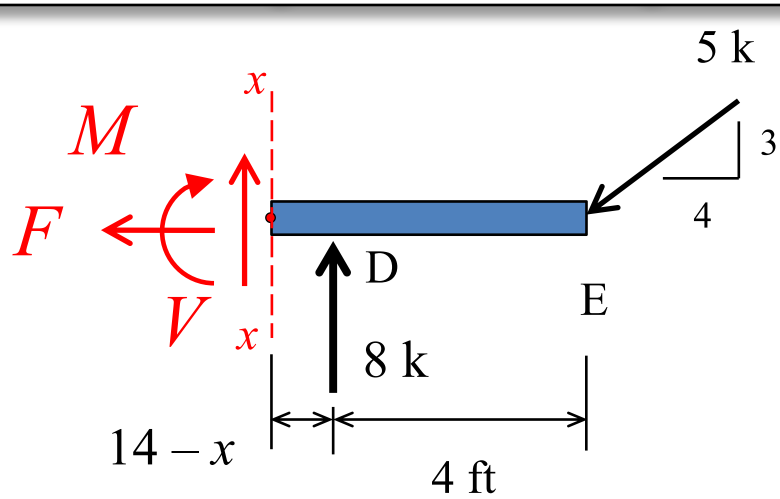


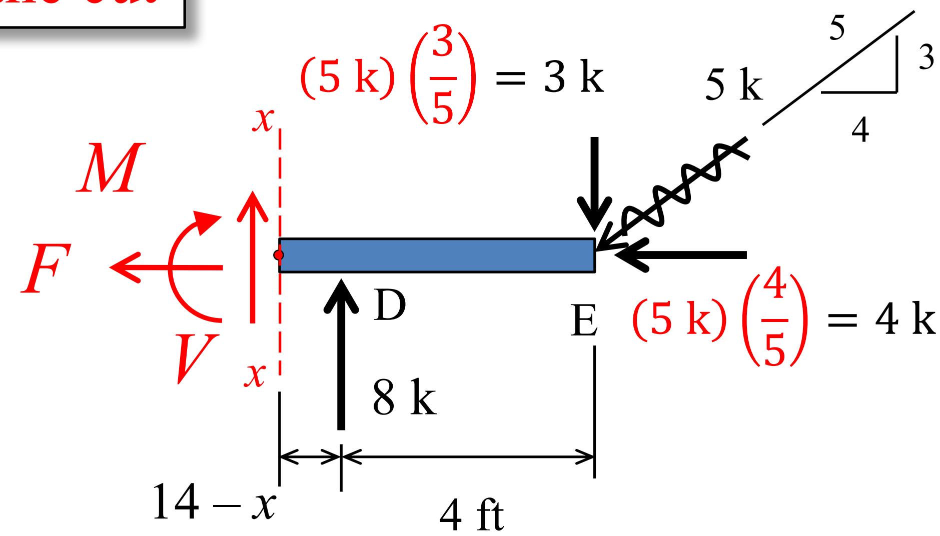
Can apply equations of equilibrium to either FBD to find internal forces

Let's use the section to the right of the cut

Find Internal Forces Between Points C and D

FBD of segment to the right of the cut





$$+\uparrow \sum F_y = 0$$

$$11 \leq x \leq 14$$
$$V(11) = V_C = -5\text{ k}$$
$$V(14) = V_D = -5\text{ k}$$

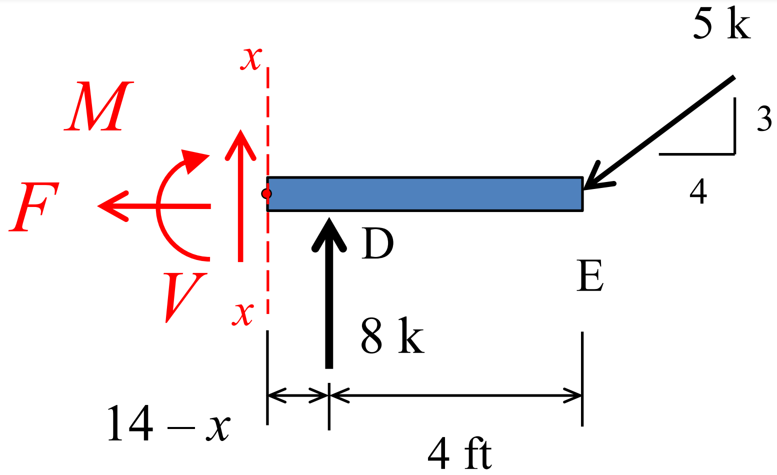
$$\xrightarrow{+} \sum F_x = 0$$

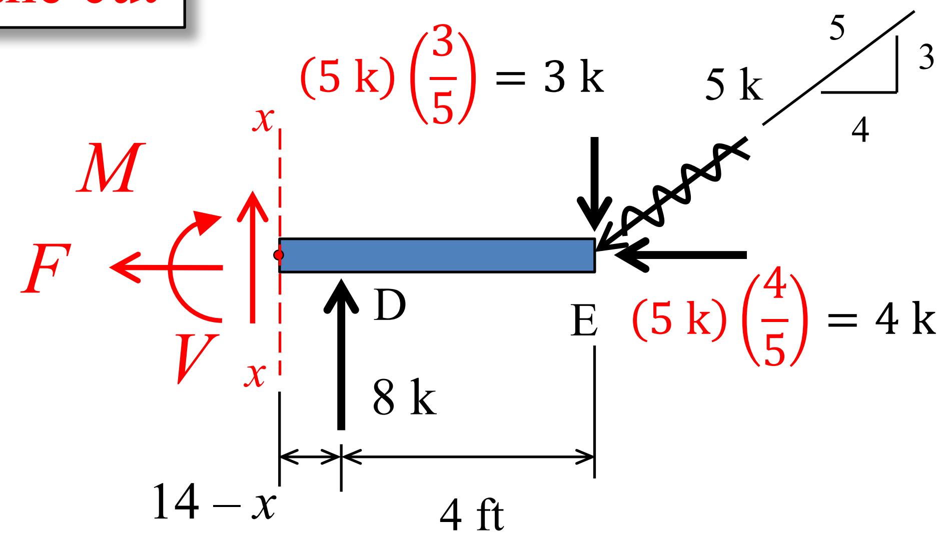
$$V = -5\text{ k}$$

F = -4 k

Find Internal Forces Between Points C and D

FBD of segment to the right of the cut



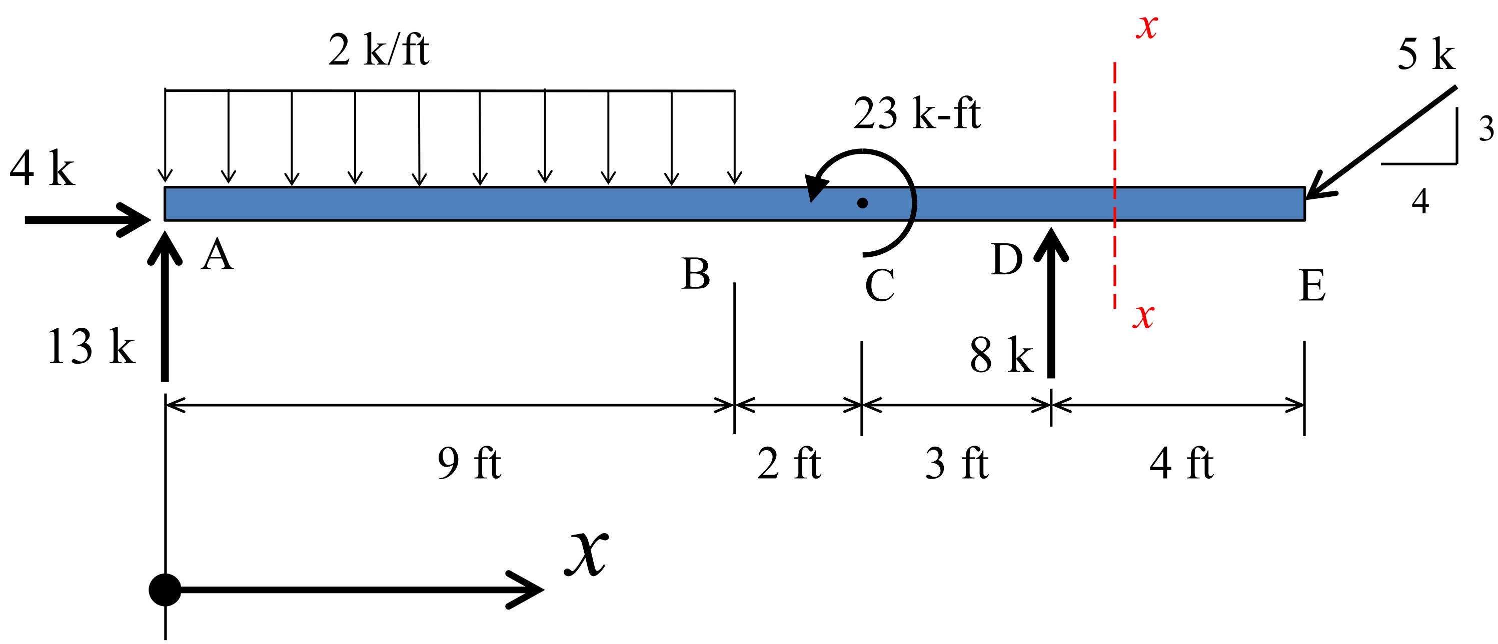


$$\circlearrowleft + \sum M_x = 0$$

$$11 \le x \le 14$$
$$M(11) = M_C = 3\text{ k–ft}$$
$$M(14) = M_D = -12\text{ k–ft}$$

$$M = -5x + 58$$

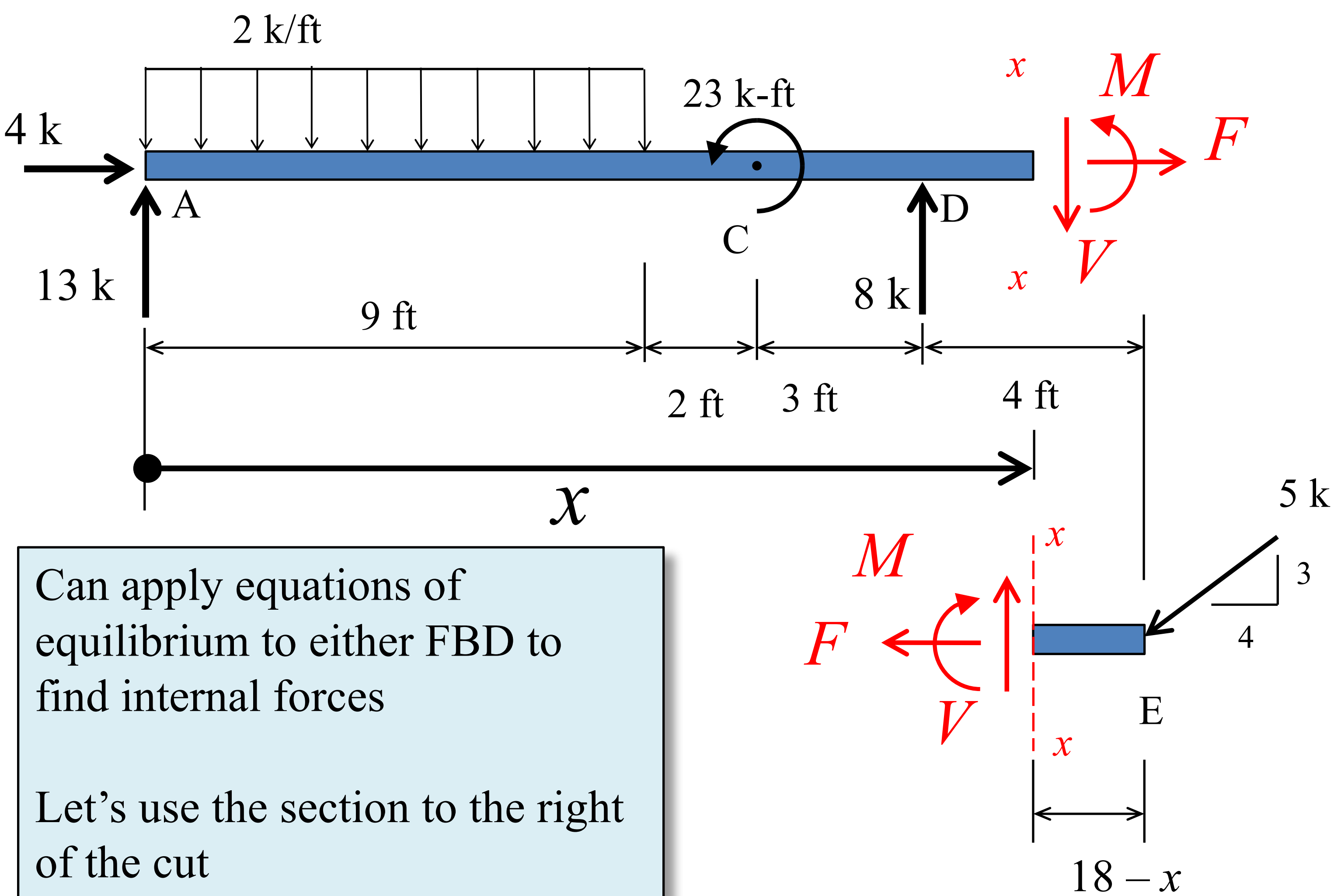
Find Internal Forces Between Points D and E



Next, cut the beam at an arbitrary point between points D and E

Find Internal Forces Between Points D and E

FBDs of Sections on either side of the cut

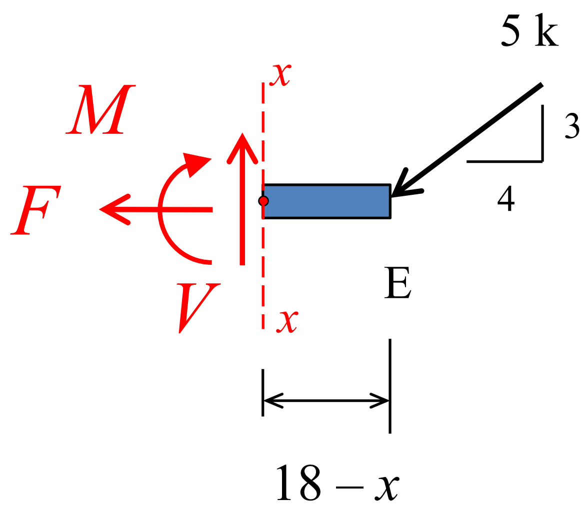


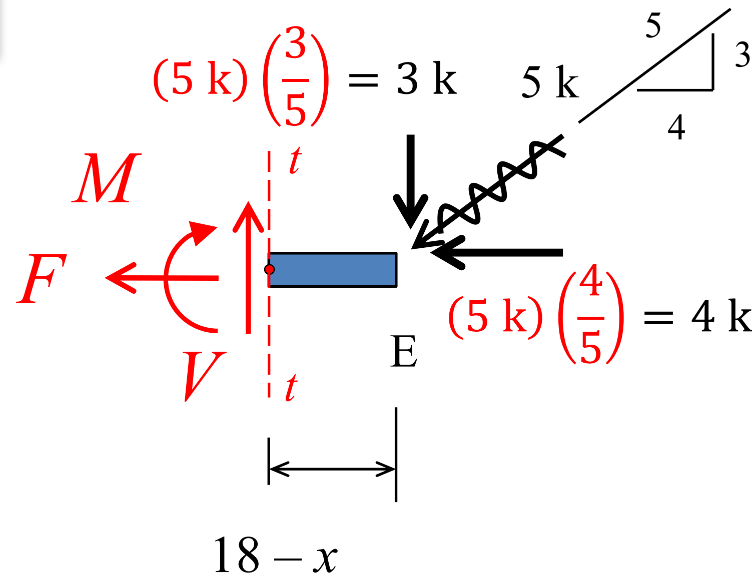
Can apply equations of equilibrium to either FBD to find internal forces

Let's use the section to the right of the cut

Find Internal Forces Between Points D and E

FBD of segment to the right of the cut





$$+\uparrow \sum F_y = 0$$

$$\xrightarrow{+} \sum F_x = 0$$

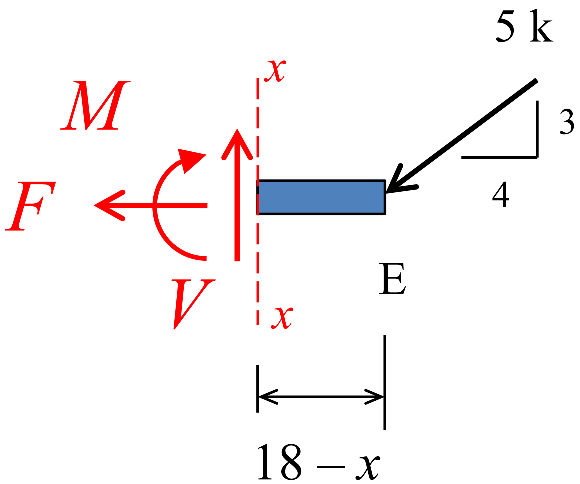
$$14 \le x \le 18$$
$$V(14) = V_D = 3\text{k}$$
$$V(18) = V_E = 3\text{ k}$$

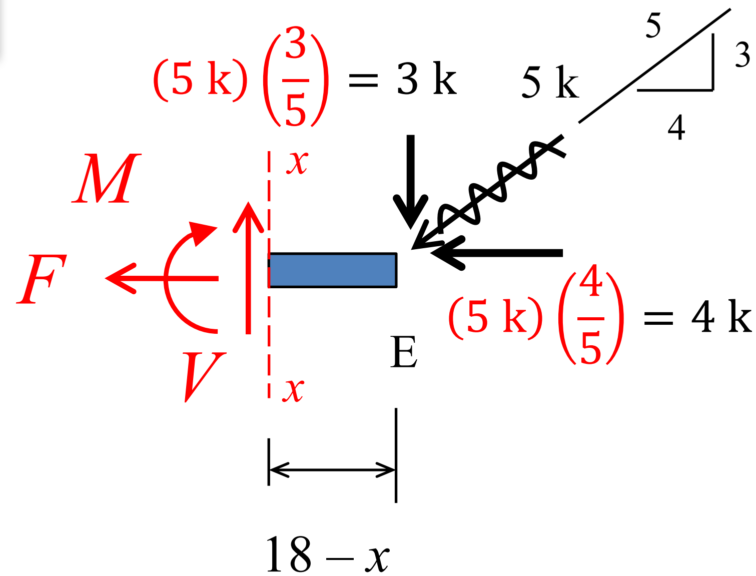
$$V = 3\text{ k}$$

F = -4 k

Find Internal Forces Between Points D and E

FBD of segment to the right of the cut





$$\circlearrowleft + \sum M_x = 0$$

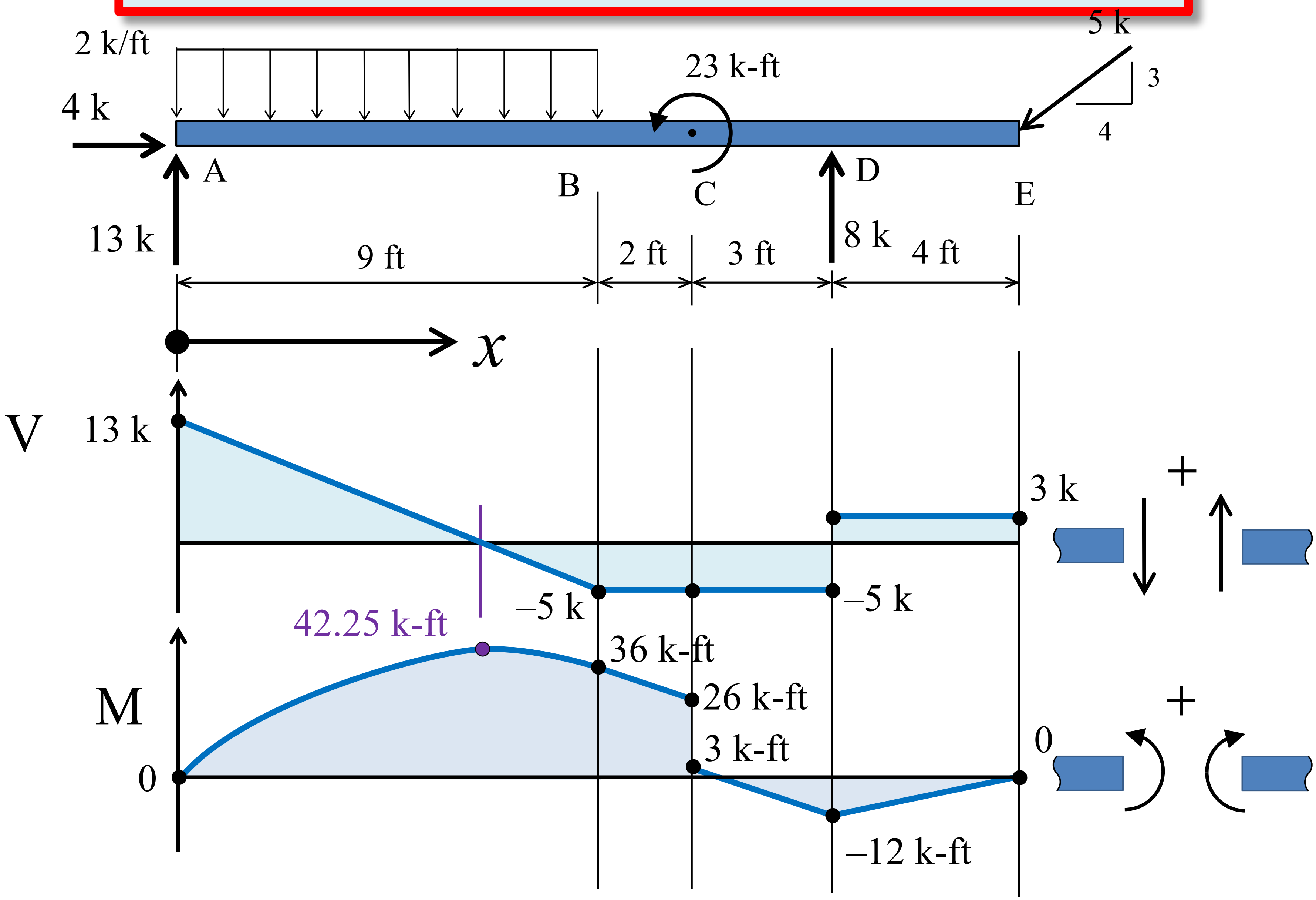
$$14 \leq x \leq 18$$

$$M(14) = M_D = -12 \text{ k–ft}$$

$$M(18) = M_E = 0$$

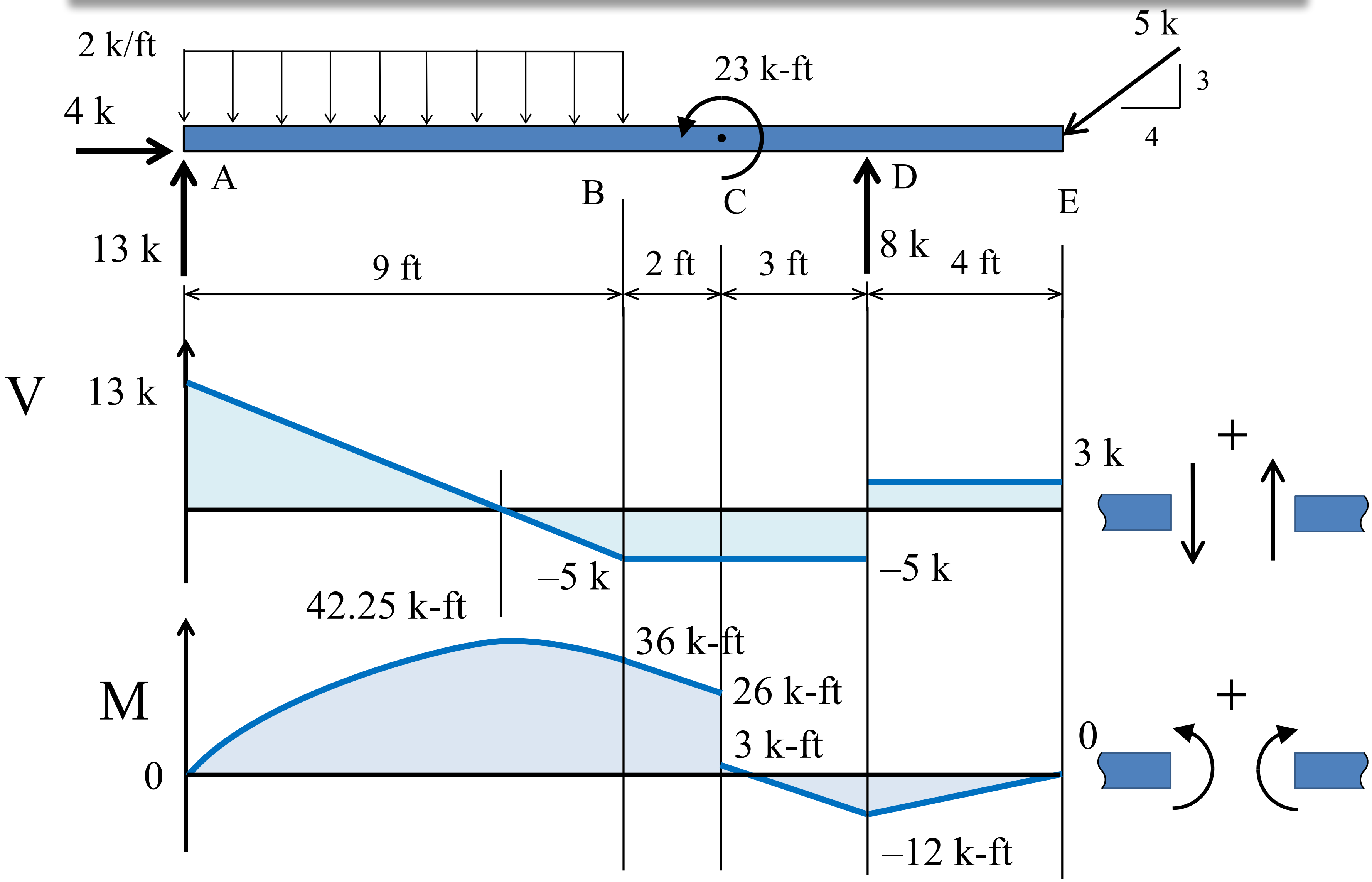
$$M = 3x - 54$$

Plot the V and M Functions for the Beam



5. Check each diagram for errors;

- Check discontinuities at location of applied forces in shear diagram,
- Check discontinuities at location of applied moment in moment diagram.



Previous Results are Points on the V and M Diagrams